A Method for Multiple Fault Diagnosis in Analog Circuits

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Abstract----Fault diagnosis of analog circuits is essential for analog and mixed-signal circuits testing and maintenance. In this paper, a new method for multiple fault diagnosis in linear analog circuits is proposed based on the large change sensitivity analysis and ambiguity group locating technique. Test equation establishes the relationship between the measured responses and deviations of faulty parameters. Multiple excitations and corresponding measurements are required for fault location. The parameter evaluation can provide the exact parameter solutions. The faulty parameter deviations can be between zero and infinity. The proposed method is extremely effective for the circuit with very limited accessible nodes and is also computationally efficient.

I. INTRODUCTION

There is an urgent need for efficient fault diagnosis tools to test analog and mixed-signal circuits, caused by a tremendous growth in design complexity and reduced access to analog parts. System-on-chip solutions favored in modern microelectronic industries compound analog testing problem. Assuming that the circuit topology and the nominal values of circuit parameters are known, fault diagnosis is to obtain the information about the faulty parameters inside the circuit based on the analysis of the limited response measurements. Fault detection, fault location and parameter evaluation are three dominant tasks of the fault diagnosis. Due to the inherited features of analog circuits such as nonlinearity, component tolerances and limited accessibility, the analog fault diagnosis techniques lag far behind their counterparts – the digital fault diagnosis techniques, in which the fault models are well established and the computer-aided diagnosis is extremely efficient. Since the 1970s, many methodologies and techniques for analog fault diagnosis have been proposed [1-2]. Today it is still one of the most challenging topics among testing engineers and academic researchers [3-10].

The multiple fault diagnosis techniques are less developed than the single fault diagnosis because it is more difficult to model and detect multiple faults, particularly in the presence of tolerance or measurement noise. In addition, in multiple fault situation one fault's effect on the circuit could be masked by the effects of the other faults. So the efforts of exploring multiple fault diagnosis techniques are still in process [7-10]. For practical reasons, the number of measurements is usually less than the number of nodes or parameters, but it is assumed to be greater than the number of faulty parameters of the tested circuit. Facing up these practical problems, a new multiple fault diagnosis method is proposed in this paper. Test equation relates the measured responses of a faulty circuit with the faults inside the circuit by a constant coefficient matrix. Fault location is implemented by a recently developed approach: location of ambiguity groups with the test equation in its minimum form. The parameter evaluation is accomplished by analyzing the test equation. The proposed method generalizes the single fault diagnosis approach, which was recently reported in a journal paper [6].

II. TEST EQUATION

Assume that the circuit under test (CUT) has *n+1* nodes and *p* parameters. The modified nodal equations [11] for the faulty circuit and fault-free circuit under the same excitations are as follows:

$$T_0 X_0 = W_0 \ , \tag{1}$$

$$TX = (T_0 + \Delta T)(X_0 + \Delta X) = W_0 \ , \tag{2}$$

where T_0 and T are *nxn* coefficient matrices, X_0 and X are *nx1* vectors of node voltage and/or parameter currents, W_0 is a *nx1* excitation vector, ΔT and ΔX are deviations of the coefficient matrix and the solution vector from their nominal values.

Suppose that f of p parameters are faulty and changed from their nominal values $h_{10}, h_{20}, \ldots, h_{f0}$ to the new values $h_1 = h_{10} + d_1,\ h_2 = h_{20} + d_2, \ldots,\ h_f = h_{f0} + d_f$, where $d_1, d_2, \ldots, d_f$ are the parameter deviations and deviation vector d is an *fx1* vector:

$$d = [d_1 \ d_2 \ \ldots \ d_f]^t \ , \tag{3}$$

Here the superscript t represents the transpose of vector or matrix. The corresponding changes in the coefficient matrix are in the form $p_n d_v q_n{}^t$ with

$$\begin{aligned} p_n &= e_{n_i} - e_{n_j} \\ q_n &= e_{n_k} - e_{n_l} \qquad n = 1, 2, \ldots, f \ , \end{aligned} \tag{4}$$

where e_n represents a *nx1* vector of zeros except for the v^{th} entry, which is equal to one.

The admittance matrix for the faulty circuit now has the following form:

$$\begin{aligned} T &= T_0 + \Delta T = T_0 + \sum_{n=1}^{f} p_v d_n q_n^t \\ &= T_0 + P \, diag(d) \, Q^t \end{aligned} \tag{5}$$

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Here $diag(d)$ is an *fxf* diagonal matrix and *P* and *Q* are *nxf* matrices which contain 0 and ± 1 entries:

$$P = [p_1 \;\; p_2 \;\; \dots \; p_f \,] \qquad (6)$$
$$Q = [q_1 \;\; q_2 \;\; \dots \; q_f \,]$$

Define *F* as the faulty parameter set, and assume that the faulty parameter $F_v \; (v = 1, 2, \dots, f)$ is located on intersection of rows i_v and j_v and columns k_v and l_v of coefficient matrix *T*, then matrix *P* and *Q* in (6) have the following forms considering (4):

$$P = [e_{i_1} - e_{j_1} \quad e_{i_2} - e_{j_2} \quad \dots \quad e_{i_f} - e_{j_f}\,]$$
$$Q = [e_{k_1} - e_{l_1} \quad e_{k_2} - e_{l_2} \quad \dots \quad e_{k_f} - e_{l_f}\,] \qquad (7)$$

Re-write the vector X_0 in following form:

$$X_0 = [x_{01} \;\; x_{02} \; \dots \; x_{0n}]^t, \qquad (8)$$

Thus the product of Q^t and X_0 can be written as

$$\begin{aligned} Q^t X_0 &= [e_{k_1} - e_{l_1} \quad e_{k_2} - e_{l_2} \quad \dots \; e_{k_f} - e_{l_f}\,]^t X_0 \\ &= [\,x_{k_1} - x_{l_1} \;\; x_{k_2} - x_{l_2} \; \dots \; x_{k_f} - x_{l_f}\,]^t \qquad (9) \\ &= [\,x_{k_1 l_1} \;\;\; x_{k_2 l_2} \;\; \dots \;\; x_{k_f l_f}\,]^t \end{aligned}$$

Applying the Woodbury formula [12] in matrix theory

$$\left(A + PS^{-1}V\right)^{-1} = A^{-1} - A^{-1}P\left(S + VA^{-1}P\right)^{-1}VA^{-1} \qquad (10)$$

to (5) with $A{=}T_0$, $S^{-1} = diag(d)$, $P{=}P$ and $V = Q^t$, the inverse of coefficient matrix *T* thus has the following form:

$$\begin{aligned} T^{-1} &= \left(T_0 + P\,diag(d)\,Q^t\right)^{-1} \\ &= T_0^{-1} - T_0^{-1}P\left(diag(d^{-1}) + Q^t T_0^{-1}P\right)^{-1} Q^t T_0^{-1} \end{aligned} \qquad (11a)$$

Let us define

$$b = [b_1 \;\; b_2 \; \dots b_n]^t = T_0^{-1}P$$
$$g = Q^t T_0^{-1} P,$$

then (11a) has following form

$$T^{-1} = T_0^{-1} - b\left(diag(d^{-1}) + g\right)^{-1} Q^t T_0^{-1} \qquad (11b)$$

Assume that coefficient matrices T_0 and *T* are non-singular. The solution vector *X* is then solved using (2) and considering (1) and (11b):

$$\begin{aligned} X &= T^{-1}W_0 \\ &= T_0^{-1}W_0 - b\left(diag(d^{-1}) + g\right)^{-1} Q^t T_0^{-1} W_0 \qquad (12) \\ &= X_0 - b\left(diag\left(d^{-1}\right) + g\right)^{-1} Q^t X_0 \end{aligned}$$

Thus, the deviation vector ΔX can be obtained by (12) considering (9):

$$\begin{aligned} \Delta X &= X - X_0 \qquad (13) \\ &= -b\left(diag\left(d^{-1}\right) + g\right)^{-1} Q^t X_0 \\ &= \begin{bmatrix} a_{11} & a_{12} & \dots a_{1f} \\ a_{21} & a_{22} & \dots a_{2f} \\ \dots\dots & & \\ a_{n1} & a_{n2} & \dots a_{nf} \end{bmatrix} \begin{bmatrix} x_{k_1 l_1} \\ x_{k_2 l_2} \\ \dots \\ x_{k_f l_f} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} \begin{bmatrix} x_{k_1 l_1} \\ x_{k_2 l_2} \\ \dots \\ x_{k_f l_f} \end{bmatrix}, \end{aligned}$$

Suppose the i^{th} node is accessible for measurement, then by (13):

$$\Delta X_i = [a_{i1} \;\; a_{i2} \; \dots a_{if}\,][\,x_{k_1 l_1} \;\; x_{k_2 l_2} \;\; \dots \;\; x_{k_f l_f}\,]^t \qquad (14)$$

Applying the same excitations to the CUT with different locations, we obtain

$$\begin{aligned} \Delta X_i^{(1)} &= [a_{i1} \;\; a_{i2} \; \dots a_{if}\,][\,x^{(1)}{}_{k_1 l_1} \;\; x_{k_2 l_2}{}^{(1)} \; \dots \; x_{k_f l_f}{}^{(1)}]^t \\ \Delta X_i^{(2)} &= [a_{i1} \;\; a_{i2} \; \dots a_{if}\,][\,x^{(2)}{}_{k_1 l_1} \;\; x_{k_2 l_2}{}^{(2)} \; \dots \; x_{k_f l_f}{}^{(2)}]^t \qquad (15) \\ &\dots\dots \\ \Delta X_i^{(m)} &= [a_{i1} \;\; a_{i2} \; \dots a_{if}\,][\,x^{(m)}{}_{k_1 l_1} \;\; x_{k_2 l_2}{}^{(m)} \; \dots \; x_{k_f l_f}{}^{(m)}]^t \end{aligned}$$

or

$$\begin{aligned} \Delta X^M{}_i &= \begin{bmatrix} \Delta X_i^{(1)} \\ \Delta X_i^{(2)} \\ \dots\dots \\ \Delta X_i^{(m)} \end{bmatrix} = \begin{bmatrix} x^{(1)}{}_{k_1 l_1} & x_{k_2 l_2}{}^{(1)} & \dots & x_{k_f l_f}{}^{(1)} \\ x^{(2)}{}_{k_1 l_1} & x_{k_2 l_2}{}^{(2)} & \dots & x_{k_f l_f}{}^{(2)} \\ \dots\dots & & & \\ x^{(m)}{}_{k_1 l_1} & x_{k_2 l_2}{}^{(m)} & \dots & x_{k_f l_f}{}^{(m)} \end{bmatrix} \begin{bmatrix} a_{i1} \\ a_{i2} \\ \dots \\ a_{if} \end{bmatrix} \qquad (16) \\ &= X_b{}^{MF} a_i \end{aligned}$$

where *M* is the node set for excitations and *m* is the number of different excitations and measurements. Assume that $f{+}1{<}m{<}p$, then the coefficient matrix $X_b{}^{MF}$ has more rows than columns. Note that the vector a_i is invariant with the same excitation sources although different locations according to its definition in (13). Equation (16) establishes the relationship between the measured responses of faulty circuit $X_b{}^{MF}$ and the faulty parameter deviations d, since according to (13) vector a_i is a linear function of d. Then (16) is called **test equation**.

III. FAULT DIAGNOSIS

Fault detection is easily judged by the measurement vector ΔX_i^M: if ΔX_i^M is a zero vector, it is concluded that no faults are detected under the existing measurement vector; Otherwise, it is concluded that at least one faulty parameter is detected.

Next we will locate the faulty elements, i.e., the exact position of $F_v \; (v = 1, 2, \dots, f)$ among the *p* parameters of the faulty circuit.

Let us define $\qquad X_b{}^{MP} = \begin{bmatrix} x_{b_1}{}^{(1)} & x_{b_2}{}^{(1)} & \dots & x_{b_p}{}^{(1)} \\ x_{b_1}{}^{(2)} & x_{b_2}{}^{(2)} & \dots & x_{b_p}{}^{(2)} \\ & \dots\dots & & \\ x_{b_1}{}^{(m)} & x_{b_2}{}^{(m)} & \dots & x_{b_p}{}^{(m)} \end{bmatrix}$

to be the matrix including all of the nominal branch voltages and/or parameter currents of the fault-free circuit. The columns of matrix X_b^{MP} correspond to the circuit parameters. Since measurement vector ΔX_i^M and the nominal matrix X_b^{MP} are known, we construct a new matrix B_S by concatenating the vector ΔX_i^M with matrix X_b^{MP} :

$$B_S = [\Delta X_i^M \quad X_b^{MP}] \tag{17}$$

Now the location of faulty parameters will be transferred to a mathematical problem: to locate the minimum size ambiguity group in matrix B_S which satisfies (16) by checking the linear dependence relationship between ΔX_i^M and the columns of X_b^{MP} . One obvious way is to have a combinatorial search which requires the number of operations $O\left(\binom{p}{f}\right)$ [2]. More efficient methods are expected to reduce the computation cost. A recent research result which requires the number of operation $O(p^3)$ can be utilized to implement such purpose [9-10]. Thus we can locate the faulty elements F_v $(v = 1, 2, \ldots, f)$, i.e., the exact positions of row i_v and j_v , columns k_v and l_v in the coefficient matrix. The proposed method for multiple fault diagnosis in [10] is based on the nodal analysis and the faulty current nodes are first located, and then the faulty parameters are located by using incident signal matrix. The method proposed in this paper is based on the large change sensitivity analysis, and the faulty parameters are located directly. Both proposed methods utilize the similar ambiguity groups locating technique with different test equations.

The invariant vector a_i can be computed by solving (16):

$$a_i = \left(\left(X_b^{MF}\right)^t X_b^{MF}\right)^{-1} \left(X_b^{MF}\right)^t \Delta X_i^M \tag{18}$$

Then, the deviation vector d can be exactly computed by

$$d = a_i \;\; rdivide\left(-b_i - a_i g\right) \tag{19}$$

where *rdivide* is an element-by-element division performance of two vectors.

IV. EXAMPLE CIRCUIT

Le us apply the proposed method to an example circuit shown in Fig. 1 with 21 nodes and 39 resistors. Nominal values of circuit parameters are as follows (all resistors in Ω): R1=2.125, R2=3.6, R3=4.7, R4=11.5, R5=12.6, R6=21.2, R7=3.7, R8=0.54, R9=3.54, R10=3.125, R11=6.6, R12=5.7, R13=19.5, R14=12.8, R15=12.2, R16=3.2, R17=1.54, R18=8.7, R19=2.27, R20=3.16, R21=41.7, R22=31.5, R23=22.6, R24=51.2, R25=13.7, R26=3.44, R27=13.4, R28=31.9, R29=16.1, R30=11.7, R31=11.5, R32=17.8, R33=22.2, R34=23.2, R35=11.4, R36=18.7, R37=3.12, R38=33.2, R39=8.67. The unique current source J is applied to nodes {0, 1} with unit amplitude, $J = 1A$ as demonstrated in Fig. 1.

Assume that there are two faulty parameters: R9 is changed from $3.54\,\Omega$ to $7.9\,\Omega$ and R37 is changed from $3.12\,\Omega$ to $2.8\ \Omega$. The corresponding admittance deviations are $\Delta G9 = 1/7.9 - 1/3.54 = -0.1559/\Omega$ and $\Delta G37 = 1/2.8 - 1/3.12 = 0.03663/\Omega$. The node {2} is selected as the only measurement node. The unit current source is applied to nodes {2, 4, 15, 16, 17} respectively and the corresponding nodal voltage at node {2} is measured. Thus *n=20*, *p=39*, *f=2*, *m=5* and *f+1<m<p*. The measured changes of nodal voltage are:

$$\Delta X^M = \begin{bmatrix} 8.9005\text{e-}001 \\ 9.1400\text{e-}001 \\ 3.6510\text{e-}002 \\ 3.2306\text{e-}002 \\ 3.8445\text{e-}002 \end{bmatrix}$$

which indicates the fault(s) detected in the CUT.

Apply the ambiguity group locating technique in [10] to the test equation, a *4x35* matrix *C* is obtained after Gaussian elimination step and QR factorization step. The column permutation is {39, 15, 2, 35, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 3, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 4, 36, 37, 38, 1}. Thus the basis is {39, 15, 2, 35} and co-basis is {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 3, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 4, 36, 37, 38, 1}.

According to Lemma 2 in [10], only one suspicious faulty group *F*={39, 15, 35, 9} is qualified with parameter {9} from the co-basis and parameter {39, 15, 35} from the basis. The current minimum size of qualified *F* is 4.

Parameter {9} from the co-basis is swapped with the parameter {39} from the basis according to the swapping procedures in [10]. A new matrix *C* results. Re-apply Lemma 2 in [10] to the new matrix *C*, 5 qualified suspicious faulty groups are obtained: F={9, 2, 35, 5}, F={9, 15, 35, 39}, F={9, 15, 35, 36}, F={9, 15, 38} and F={9, 37}. Obviously, *F*={9, 37} is the unique solution with the minimum size equal to 2. Since no smaller size of faulty set *F* can be found by swapping, thus *F*={9, 37} is the only solution located by the procedures in section III of [10] which is the exact solution for the given CUT.

Equation (16) thus has the following form:

$$\begin{bmatrix} 8.9005\text{e} - 001 \\ 9.1400\text{e} - 001 \\ 3.6510\text{e} - 002 \\ 3.2306\text{e} - 002 \\ 3.8445\text{e} - 002 \end{bmatrix} = \begin{bmatrix} 1.8790\text{e} + 000 & 2.2168\text{e} - 002 \\ 1.9296\text{e} + 000 & 2.0659\text{e} - 002 \\ 7.9465\text{e} - 002 & 1.1619\text{e} + 000 \\ 7.2322\text{e} - 002 & 2.0046\text{e} + 000 \\ 8.3425\text{e} - 002 & 1.1018\text{e} + 000 \end{bmatrix} a_i$$

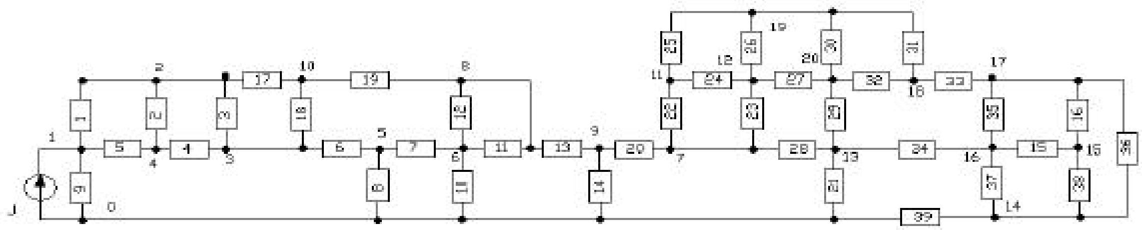

Figure 1. Resistive network example

and by (18),

$$a_i = \begin{bmatrix} 4.7369\,e-001 \\ -9.7400\,e-004 \end{bmatrix}$$

Finally the corresponding parameter deviation values computed by (19) are

$$\begin{bmatrix} \Delta G9 \\ \Delta G37 \end{bmatrix} = \begin{bmatrix} -1.5590e-001 \\ 3.6630e-002 \end{bmatrix}$$

which are the exact deviation values of the faulty elements R9 and R37.

V. CONCLUSIONS

Fault diagnosis presented in this paper is to obtain the information about the faulty circuit based on the limited measured responses of the faulty circuit. The circuit topology and nominal values of circuit parameters are known , and the number of measurements is less than the number of nodes or parameters, but greater than the number of faulty parameters plus one. A new method proposed in this paper is used to detect, locate the multiple faults of the linear analog circuit, then to exactly evaluate the faulty parameter deviations. Applying the Woodbury formula in the matrix theory to the large change sensitivity approach, test equation is constructed to establish the relationship between the measured responses and the faulty parameter deviations. Fault location is implemented by a newly proposed approach: location of the minimum size ambiguity group in the test equation. Parameter evaluation is then performed from results of the test equation analysis.

One measurement node is sufficient for the proposed method although distinct excitations and measurements of accessible node voltages for faulty circuit are required for fault location. The proposed method is extremely effective for large parameter deviations and a very limited number of accessible test nodes used for excitations and measurements. The computation cost for the fault location is in the number of operations $O(p^3)$. It is computationally efficient comparing with the combinatorial search traditionally used in fault verification methods which requires the number of operations $O\left(\binom{p}{f}\right)$. A single fault diagnosis method recently reported in a journal paper [6] can be seen as a special case of the proposed method. Finally an example circuit is used to illustrate the proposed method.

VI. REFERRENCES

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