

A New Approach to Multiple Fault Diagnosis in Linear Analog Circuits

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Abstract—A new approach to multiple fault diagnosis in linear analog circuits is proposed in this paper combining the modified nodal analysis with the QR factorization technique. Fault diagnosis equation establishes the relationship between the measured responses and faulty parameter deviations. Multiple excitations are required for fault location and the number of excitations is no less than the number of faults, but less than the number of measurements. Exact circuit parameter values can be obtained in the parameter evaluation stage. An example circuit is provided finally.

I. INTRODUCTION

With the rapid development of analog VLSI chips and mixed-signal systems, the cost for testing and maintenance of such circuits is ever increasing part of the product cost and operation cost. Cost effective approach for testing and diagnosis of analog and mixed-signal circuits must be automated. Comparing with the highly automated electronic design methods which are popular in today's industry, the automated level for analog and mixed-signal circuit testing and diagnosis lags behind. The reason lies in the inherent features of analog circuits such as nonlinearity, parameter tolerances, and limited accessibility. Thus far, many efforts from industry engineers and academic researchers are devoted to this area. Several good reviews appeared in 1979 [1], 1985 [2] and 1991 [3]. Examples of research efforts after 1991 can be found in [4-6].

Among analog fault diagnosis approaches, multiple fault diagnosis techniques are less developed than the single fault diagnosis. Due to lack of efficient model and fault masking, multiple fault diagnosis in analog circuits is still a challenging topic today [7-9]. In this paper, a new approach to multiple fault diagnosis in linear analog circuits is proposed which combines the modified nodal analysis with the QR factorization. The circuit topology and the nominal values of circuit parameters are assumed known. Multiple excitations are required for the fault location and the number of excitations are required to be no less than the number of faulty parameters. The number of measurements on nodal voltages must be greater than the number of excitations. Fault diagnosis equation is established in Section II based on the modified nodal analysis. The coefficient matrix which is derived from the nominal values of circuit parameters relates the measured circuit responses to the faulty parameters. In Section III, a recently developed approach [10-11] is applied to locate the faulty parameters by applying the QR

factorization technique to the fault diagnosis equation. Fault parameter deviations are exactly obtained by analyzing the fault diagnosis equation. The proposed approach is applied to an example circuit in Section IV and conclusions are drawn in Section V.

II. FAULT DIAGNOSIS EQUATION

We only consider the linear analog circuits in this paper. Assume that the circuit under test has $n+1$ nodes and p parameters in the form of impedance Z_v ($v=1, 2, \dots, p$). Suppose the circuit parameter Z_v is described by two-port like model: its controlled port is located between nodes i_v and j_v while its controlling port is located between nodes k_v and l_v . Thus, the circuit topology can be described by two $n \times p$ matrices P and Q which is defined as follows [12]:

$$\begin{aligned} P &= [e_{i_1} - e_{j_1} \quad e_{i_2} - e_{j_2} \quad \dots \quad e_{i_p} - e_{j_p}] \\ Q &= [e_{k_1} - e_{l_1} \quad e_{k_2} - e_{l_2} \quad \dots \quad e_{k_p} - e_{l_p}] \end{aligned} \quad (1)$$

where e_n represents a $n \times 1$ vector of zeros except for the v^{th} entry, which is equal to one.

Suppose that there are e different excitations to the fault-free and faulty circuits, apply Kirchhoff current law to the fault-free circuit:

$$PI_{b0} = J, \quad (2)$$

where I_{b0} is a $p \times e$ matrix of branch currents and J is a $n \times e$ matrix of excitations,

Apply Kirchhoff voltage law to the fault-free circuit:

$$Q^T V_{n0} - Z I_{b0} = 0, \quad (3)$$

where superscript T denotes transpose of matrix, V_{n0} is a $n \times e$ matrix of nodal voltages, Z is a $p \times p$ diagonal matrix of impedances:

$$Z = \text{diag}(Z_v) \quad (4)$$

Combining (2) and (3), we have

$$\begin{bmatrix} 0 & P \\ Q^T & -Z \end{bmatrix} \begin{bmatrix} V_{n0} \\ I_{b0} \end{bmatrix} = \begin{bmatrix} J \\ 0 \end{bmatrix}, \quad (5)$$

Assume that there are f of p faulty parameters in the faulty circuit with $f \leq e$. Correspondingly, the equation for the faulty circuit is as follows:

$$\left(\begin{bmatrix} 0 & P \\ Q^T & -Z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \Delta Z \end{bmatrix} \right) \left(\begin{bmatrix} V_{n0} \\ I_{b0} \end{bmatrix} + \begin{bmatrix} \Delta V_n \\ \Delta I_b \end{bmatrix} \right) = \begin{bmatrix} J \\ 0 \end{bmatrix}, \quad (6)$$

where ΔZ is a $p \times p$ diagonal matrix of parameter deviations, ΔV_n is a $n \times e$ matrix of nodal voltage deviations, and ΔI_b is a $p \times e$ matrix of branch current deviations.

Denote

$$\begin{bmatrix} V_n \\ I_b \end{bmatrix} = \begin{bmatrix} V_{n0} \\ I_{b0} \end{bmatrix} + \begin{bmatrix} \Delta V_n \\ \Delta I_b \end{bmatrix}, \quad (7)$$

Equation (6) can be simplified after considering (5):

$$\begin{bmatrix} 0 & P \\ Q^T - Z \end{bmatrix} \begin{bmatrix} \Delta V_n \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} 0 \\ -\Delta Z I_b \end{bmatrix}, \quad (8)$$

Suppose that m nodal voltages are measured with $e < m$, then (8) can be decomposed as follows:

$$\begin{bmatrix} 0 & 0 & P \\ Q_1^T & Q_2^T & -Z \end{bmatrix} \begin{bmatrix} \Delta V_n^M \\ \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} 0 \\ -\Delta Z I_b \end{bmatrix}, \quad (9)$$

where M denotes the set of measured nodes and N denotes the set of all nodes, Q_1 is a $n \times m$ matrix and Q_2 is a $n \times (p-m)$ matrix.

Move the measured part to the right of (9) to obtain the following form:

$$\begin{bmatrix} 0 & P \\ Q_2^T - Z \end{bmatrix} \begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} 0 \\ -\Delta Z I_b \end{bmatrix} + \begin{bmatrix} 0 \\ -Q_1^T \Delta V_n^M \end{bmatrix}, \quad (10)$$

Equation (10) which is called **fault diagnosis equation** relates the measured responses deviations ΔV_n^M with the faulty parameter deviations ΔZ . The left-hand side $(n+p) \times (n+p-m)$ coefficient matrix of (10) can be constructed from the circuit topology and nominal values of circuit parameters. The solution matrix of (10) has a size of $(n+p-m) \times e$. The right side of (10) is a $(n+p) \times e$ matrix with f unknown entries due to faulty parameters. Thus, $(n+p-f) \times e$ linear equations with $(n+p-m) \times e$ variables can be obtained from (10). Since $m > f$, solution to (10) can be uniquely determined.

III. FAULT DIAGNOSIS PROCESS

If ΔV_n^M is not a zero matrix, it is concluded that at least one faulty parameter is detected by the measurement matrix ΔV_n^M . Then we will locate the position of faulty parameters in the faulty circuit by checking the dependency relationship between the rows of the coefficient matrix of (10) and the rows of the right side of (10).

Construct a $(n+p+e-m) \times (n+p)$ new matrix by appending the coefficient matrix of (10) to the second item of the right side of (10):

$$B = \begin{bmatrix} 0 & 0 & P \\ -Q_1^T \Delta V_n^M & Q_2^T & -Z \end{bmatrix}^T, \quad (11)$$

The columns of matrix B correspond to the circuit nodes and parameters. Therefore, locating the faulty parameters in

matrix B is equivalent to identifying the independent columns in matrix B . One apparent approach is a comprehensive search which requires the number of operations $O\left(\binom{n+p}{f}\right)$. More computationally efficient approaches are expected to reduce cost. A recently developed technique can be utilized here which requires the number of operation $O(p^3)$ [10-11]:

The rank of matrix B determines the maximum number of faulty parameters that can be uniquely identified by solving (10). Because matrix B has more columns than rows, B can be written as

$$B = B_1 [I \ C] \quad (12)$$

where $(n+p+e-m) \times r$ matrix B_1 has the full column rank equal to the rank r of the matrix B , and $(n+p+e-m) \times (n+p-r)$ matrix C expands the dependent columns of B into a set of the basis columns B_1 . Thus, $n+p+e-m \leq r < n+p$.

As a result of the QR factorization of matrix B , we can formulate the following equation:

$$BE = \hat{Q}R \quad (13)$$

Where E is $(n+p) \times (n+p)$ permutation matrix with only a single nonzero element equal to one in each column, $\hat{Q}$ is $(n+p+e-m) \times (n+p+e-m)$ orthogonal matrix, and R is $(n+p+e-m) \times (n+p)$ upper triangular matrix. Since R is an upper triangular matrix with more columns than rows, R can be written as

$$R = \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} \quad (14)$$

Where R_1 is $r \times r$ upper triangular matrix and R_2 is $r \times (n+p-r)$ matrix.

Theorem [11]:

A linear combination matrix C can be numerically obtained from the QR factorization of the matrix B using

$$C = R_1^{-1} R_2 \quad (15)$$

Thus C is $r \times (n+p-r)$ matrix. Since the number of measurements m is larger, but not by far than the number of excitations, matrix C has a very limited number of columns due to $0 < n+p-r \leq m-e$. The number of rows of matrix C is relatively large comparing with its column numbers due to $r \geq n+p+e-m$.

Fault diagnosis equation (10) is a very unusual equation. It contains unknown matrix of voltage and current deviations on the left-hand side and partly unknown right-hand side. The test matrix B has the rank equal to $n+p+e-m$ (where $f \leq e$), however, the rank of

$$S = \begin{bmatrix} 0 & P \\ Q_2^T & -Z \end{bmatrix}$$

is equal to $n+p-m$. So, the increase in the rank of matrix B over the rank of matrix S is due to the presence of faulty

parameters which make part of the right-hand side of (10) independent on rows of matrix S . Therefore, all columns of matrix B which correspond to faulty parameters will be selected to the basis and (very important!) rows of matrix S which are not in the basis will be independent from these columns. This independence results in the following Lemma:

Lemma:

If all of the faulty parameters are included in the basis, then the circuit parameters corresponding to zero rows in the matrix C are faulty.

Since $f \leq e$ and the faulty parameters are independent from each other, all of the faulty parameters are guaranteed to be included in the basis. Therefore by applying Lemma to the obtained matrix C , we can identify the faulty elements directly (No search required at all!).

After the location of faulty parameters, (10) can be decomposed according to positions of faulty parameters:

$$\begin{bmatrix} 0 & P \\ Q_{21}^T & -Z_{ff} \\ Q_{22}^T & -Z_f \end{bmatrix} \begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\Delta Z_f I_b \end{bmatrix} + \begin{bmatrix} 0 \\ -Q_{11}^T \Delta V_n^M \\ -Q_{12}^T \Delta V_n^M \end{bmatrix}, \quad (16)$$

$$\text{or} \quad \begin{bmatrix} 0 & P \\ Q_{21}^T & -Z_{ff} \end{bmatrix} \begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} 0 \\ -Q_{11}^T \Delta V_n^M \end{bmatrix}, \quad (17)$$

$$\begin{bmatrix} Q_{22}^T & -Z_f \end{bmatrix} \begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = \begin{bmatrix} -\Delta Z_f I_b \end{bmatrix} + \begin{bmatrix} -Q_{12}^T \Delta V_n^M \end{bmatrix}, \quad (18)$$

Here the matrices Q_1 , Q_2 , Z , and ΔZ are decomposed into two parts: one part corresponds to faulty parameters, while another corresponds to fault-free parameters.

The solution to (17) can be uniquely determined by

$$\begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} = ((S_1)^T S_1)^{-1} (S_1)^T \begin{bmatrix} 0 \\ -Q_{11}^T \Delta V_n^M \end{bmatrix} \quad (19)$$

$$\text{where} \quad S_1 = \begin{bmatrix} 0 & P \\ Q_{21}^T & -Z_{ff} \end{bmatrix}.$$

Then, the values of branch currents in faulty circuit I_b can be obtained by (7). Re-arranging (18), we have

$$\Delta Z_f I_b = - \left(\begin{bmatrix} Q_{22}^T & -Z_f \end{bmatrix} \begin{bmatrix} \Delta V_n^{N-M} \\ \Delta I_b \end{bmatrix} + \begin{bmatrix} -Q_{12}^T \Delta V_n^M \end{bmatrix} \right) = S_2 \quad (20)$$

To reduce the computational efforts, select only one column I_{b1} from matrix I_b and only one column S_{21} from matrix S_2 . The faulty parameter deviations can be exactly computed by solving (20):

$$\Delta Z_f = S_{21} \text{ rdivide } I_{b1}, \quad (21)$$

where *rdivide* is an element-by-element division performance of two vectors. As an alternative, and to reduce effects of round-off errors, we could solve (20) by dividing S_2 by I_b element by element and taking row average to obtain ΔZ_f .

The proposed approach for multiple fault diagnosis described in [11] is based on the modified nodal analysis and the faulty current nodes are first located. Then the faulty parameters are located by using incident signal matrix. The Gaussian elimination step and swapping step are introduced in the ambiguity group locating process. The approach proposed in this paper locates the faulty parameters directly without the Gaussian elimination step and without swapping, thus reducing the computation cost. Both proposed approaches utilize the QR factorization technique with different fault diagnosis equations.

IV. AN EXAMPLE CIRCUIT

An example circuit shown in Fig. 1 with 8 nodes and 21 parameters is used to demonstrate the proposed approach. The nominal values of the cascaded FET amplifier and simplified model of FET are indicated in Fig.1. Nominal values of FET parameters are as follows: $g_m = 5 \times 10^{-3} \text{ mho}$, $g_D = 10^{-5} \text{ mho}$, $C_{GS} = 2 \text{ pF}$, $C_{GD} = 2 \text{ pF}$, and C_{DS} is neglected. The current source $J = 0.1 \sin(2\pi \times 10^5 t) \text{ A}$ is applied to nodes $\{0, 1\}$, $\{0, 4\}$ respectively. Note that current source is only applied to nodes $\{0, 1\}$ in Fig. 1.

Assume that there are two faulty parameters: $R1$ is changed from 50Ω to 500Ω and $C15$ is changed from 1 mF to 5 mF . The corresponding impedance deviations are $\Delta Z1 = 450 \Omega$ and $\Delta S_C = 2.51j \Omega$. The nodal voltages at nodes $\{1, 4, 6, 8\}$ are measured. Thus $n=8$, $p=21$, $e=2$, $f=2$, $m=4$ and $f \leq e < m$. The measured changes of nodal voltage under two distinct excitations are:

$$\Delta V_n^M = \begin{bmatrix} 4.50e+1 - 6.20e-2i & 1.57e-5 + 4.48e-9i \\ 4.50e+1 - 6.19e-2i & 1.57e-5 + 4.54e-9i \\ 1.80e-5 + 3.00e-9i & 6.38e-12 + 1.18e-14i \\ 7.17e-12 + 3.83e-13i & 2.50e-10 + 1.38e-11i \end{bmatrix}$$

which indicates the fault(s) detected inside of the circuit.

Apply the QR factorization to the fault diagnosis equation. A 27×2 matrix C is obtained with rank of $r=2$. By analyzing permutation matrix E , co-basis includes only two circuit nodes $\{4, 8\}$ and the remaining 6 nodes and 21 parameters are included in the basis.

Analyzing 27×2 matrix C , only zero rows are found which corresponding to parameters $\{R1, C15\}$. According to Lemma in Section III, since all the circuit parameters are included in the basis, parameters $\{R1, C15\}$ are concluded as faulty parameters which are the exact solution for the given circuit.

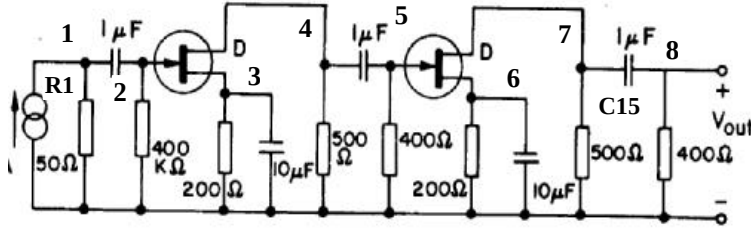
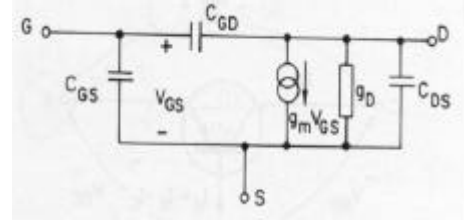


Figure 1. Amplifier example



The deviations of nodal voltages and branch currents can be obtained by (19). The branch currents and the nodal voltages in the faulty circuit can be obtained by (7). Finally, the deviations of faulty parameters are exactly evaluated using (21):

$$\begin{bmatrix} \Delta Z1 \\ \Delta sC15 \end{bmatrix} = \begin{bmatrix} 4.5000e+002 + 8.7393e-0014j \\ 2.5133e-000j \end{bmatrix}$$

which are the exact deviation values of the faulty elements R1 and C15.

V. CONCLUSIONS

Multiple fault diagnosis in analog circuit is a challenging topic for testing engineers and academic researchers. In this paper, a new multiple fault diagnosis approach for linear analog circuits is proposed to detect, locate the faults and evaluate the faulty parameters. Measured deviations of the selected nodal voltages for the fault-free and faulty circuit indicate that at least one fault was detected under the given measurement conditions. By analyzing the circuit topology and utilizing the Kirchhoff current/voltage law, fault diagnosis equation is established to relate the measured response deviations to the faulty parameter deviations. Coefficient matrix is only related to the circuit topology and nominal values of circuit parameters in the impedance form. Multiple excitations are required for the location of faulty parameters and the number of excitations should be no less than the number of faults, but less than the number of selected measurement nodes. A newly developed technique based on the QR factorization is applied in this paper to locate the faulty parameters in the fault diagnosis equation. Faulty parameter deviations can be exactly evaluated by analyzing the fault diagnosis equation after locating the faulty parameters.

The proposed approach is extremely effective for large parameter deviations and limited number of accessible test nodes used for excitations and measurements. The computation cost for the fault location is reduced comparing with the comprehensive search. Finally an example circuit is used to demonstrate the proposed approach.

VI. REFERENCES

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