Fault Diagnosis in Mixed-Signal Low Testability System

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ABSTRACT

This paper describes a new approach for fault diagnosis of analog multi-phenomenon systems with low testability. The developed algorithms include identification of ambiguity groups, fault diagnosis methodology and solving low testability equations. Our aim is to identify a minimum number of faulty parameters that satisfy fault equations called a minimum form solution. An algorithm to find a minimum form solution is presented, which is based on the solution invariant matrix and an identification of singular cofactors of this matrix. System simulation using a developed C++ and Matlab programs was performed to test different faulty circuits. Test examples are discussed and simulation results are presented.

KEY WORDS: ambiguity groups, fault diagnosis, minimum form solution, solution invariant matrix, singular cofactor

1. Introduction

Mixed-signal ICs become more and more important with the quick development in many areas such as mobile communications, process control, automotive ASIC and smart sensors [1][2][3]. Because of the driving force of the market, these products must have high quality and low cost. As a result, test costs, time, and quality become more and more important. For many years, the testability of digital IC has been studied extensively [4][5]. Standard fault models such as "stuck-at" model provide better fault coverage and less testing time than before. In addition, scan path access [6] facilitates the automatic testing. Mature commercial automated DFT tools are also available. However, because of the lack of standard analog fault model, standard mixed-signal DFT methodology and computer-aided test (CAT) tools, the design for testability (DFT) of mixed-signal IC development has lagged far behind [7][8]. Furthermore, high quality analogue tests are most expensive in terms of both test development costs and test implementation. In the commercial market, up to 80% of the test costs are due to the analogue functions that typically occupy only around 10% of the chip area.

Because of the limited testing points, parameter tolerance, and non-linearity of analog component model, fault diagnosis of analogue circuits is extremely difficult [9][10][11][12][13]. Efficient analog fault diagnosis algorithm must provide good fault coverage, low computational cost and reduce the number of test points at the same time. Furthermore, as the size of the analog integrated circuits become smaller and smaller, the accessibility for measurements steadily decreases. Defects on circuit can be classified as catastrophic (or hard) and parametric (or soft) faults [14][15]. Traditionally, there are two approaches to diagnose analog fault: simulate-before-test (SBT) and simulate-after-test (SAT) [9]. While SBT approach is often used to isolate hard faults based on fault dictionaries, SAT is more suitable for the diagnosis of soft faults depending on solving fault diagnosis equations [11][16]. These test equations and their properties are a focus point of our study.

In order to reduce testing time and cost, testability analysis is performed to select optimal measurements for the Circuit-Under-Test (CUT) [17]. Test generation can be in DC [18] or AC [19] domain using a set of linear [20] or non-linear [13] fault diagnosis equations. According to Berkowitz [21], testability analysis is strictly tied to network-element-value-solvability. A well-defined quantitative testability measure proposed by Saeks et al. [22], who combined the concept of ambiguity with the solutions for test equations in a neighborhood of almost any failure, remains to be very useful now [17][23]. Several design automation tools were developed to analyze analog system testability [24][25]. In case of low testability, although the concept of ambiguity group is extremely useful, the quality of obtained test results strongly depends on its proper usage [26]. Addressing these issues Liu et al. [27] analyzed system testability based on behavioral modeling in the presence of low testability circuits with ambiguity groups. We feel that system testability in presence of ambiguity groups in mixed-signal mixed-mode systems deserve more attention.

In low testability system, no simple solution can be found using traditional solving methods for system test equations, because they are singular. In what follows we concentrate on this kind of systems. We develop an efficient approach to identify ambiguity groups, which cause singularity of test equations. Subsequently, we develop a method to solve such singular test equations. Although a unique solution is not always possible in such systems, our method provides the best possible alternative. The method provides a unique solution for all testable components. In addition, components within an ambiguity group have unique solution under the assumption of the number of faults being smaller than the rank of the corresponding ambiguity group.

In this paper, part 2 introduced the formulation of test equations and the important characteristics of testability matrix. Six Lemmas were developed and new terminologies were defined. In part 3, the fault identification techniques we applied in low testability systems were discussed and major strategies were contained in three Lemmas. In addition, detailed steps were listed to clarify the solution procedures. The organization of computer programs was presented in part 4 and the programs were implemented in C++ and Matlab. Especially, Fig. 1 illustrates the block diagram of Matlab program. Furthermore, three examples are provided in part 5 to demonstrate the solution strategies. The conclusions are described in Part 6 and the references listed in the final pages.

2. Test Equation

2.1 Objectives

The purpose of our work is to identify ambiguity groups and provide minimum form solution for the low testability systems based on sensitivity analysis, which forms a functional testing. The first-order approximation of small sensitivity analysis linearizes the relationship between circuit response functions and circuit parameters. This is true if the deviation of the circuit parameters is small.

Minimum form solution is based on ambiguity group identification, which can be distinguished by QR factorization.

2.2 Test Equation Formulation

Because the number of independent parameter faults in a modern design is limited and the changes of the individual circuit components track each other as a result of the uniform environment in the VLSI fabrication process, therefore, it is reasonable to assume that the number of faulty parameters that have to be identified using the test equations is small and that only some of them are faulty.

Let us consider test equations

$$BP = M \tag{1}$$

where $m \times p$ testability matrix B was generated from the test equations. P and M are the changes of parameters and measurements from the nominal values, respectively. Since the number of faulty parameters is small, most of elements of vector P are zero.

For a numerical stability and a reduction of the roundoff errors the testability matrix *B* must have larger number of rows than columns. If the testability matrix has the full column rank, the tested circuit is fully testable and all the tested parameters *P* can be uniquely identified. However, for low testability system, this is not the case.

2.3 Linear Combination Matrix

Let us assume that the number of **measurements** *M* used in the **test equations** is greater than the number of tested parameters *P*. The rank of testability matrix *B* determines a maximum number of the circuit parameters that can be uniquely identified by solving the test equations. If *B* does not have the full column rank, then it can be written as

$$B = B_1 \, [I \ C_1] \tag{2}$$

where $m \times r$ matrix B_1 has the full column rank equal to the rank of the matrix *B*, and $r \times (p - r)$ matrix C_1 expand the dependent columns of *B* into a set of the basis columns of *B*. So we call C_1 a **linear combination matrix.** Selection of independent columns B_1 is not unique and is an important issue in solving the test equations in the presence of ambiguities.

Mathematically, an ambiguity group can be defined as a set of circuit parameters, which correspond to linearly dependent columns of the testability matrix *B*. Since a superset of dependent columns is also dependent, a canonical ambiguity group was defined. It is a set of parameters, which correspond to linearly dependent columns of *B*, such that every subset of these columns is linearly independent. All canonical ambiguity groups have the rank deficiency equal to one. A combination of canonical ambiguity groups with at least one common element was defined as the ambiguity cluster. Finally, all circuit components, which correspond to columns of testability matrix that are not included in any ambiguity group, are called surely testable components.

As a result of the QR factorization of $m \times p$ testability matrix *B* we can formulate the following equation:

$$BE = QR \tag{3}$$

where *E* is $p \times p$ column selection matrix, *Q* is $m \times m$ orthogonal matrix, and *R* is $m \times p$ upper triangular matrix. Matrix *E* has only a single nonzero element in each column. Each nonzero element of *E* is equal to one and the matrix product *BE* represents a permutation of the original columns of the testability matrix *B*. Matrix *R* has its rank equal to the rank of testability matrix *B*. Since *R* is an upper triangular matrix and $p<m$, therefore all rows of *R* from *p* to *m* are zero, and as a result, we need only to generate the first *p* columns of the orthogonal matrix *Q*. In our analysis we will assume that *R* was reduced to $p \times p$ matrix by removing all its zero rows.

Furthermore, in the presence of ambiguity groups in the testability matrix *B*, its rank and the rank of *R* are less than *p*. Therefore, matrix *R* can be written as

$$R = \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} \tag{4}$$

where R_1 is $r \times r$ upper triangular and has its rank equal to the rank of the testability matrix *B*.

We proved the following lemmas, which provides a basis for a numerically efficient approach to finding the ambiguity groups, ambiguity clusters and surely testable components.

Lemma 1: A linear combination matrix C_1 can be numerically obtained from the QR factorization of the testability matrix *B* using

$$C_1 = R_1^{-1} R_2 \tag{5}$$

Lemma 2: If any two columns of the linear combination matrix C_1 have simultaneously nonzero elements in at most one common row, then C_1 is in its minimum form.

Since the rank of the testability matrix is equal to a given testability measure almost everywhere in the parameter space, we will extend this result to ranks of all submatrices of the testability matrix that are used to determine the existence of the ambiguity groups. Under this assumption we may study properties of the linear combination matrix C_1 considering its equivalent binary matrix *D* that has the same size as C_1. An element of the matrix *D* is equal to one if the corresponding element of C_1 is nonzero, all other elements are set to zero. As in matrix C_1, rows of *D* correspond to the elements of the basis and columns correspond to the elements of the co-basis on a given partition. This equivalent representation simplifies the analysis of C_1 as the set theory can be used to study its structural properties.

Lemma 3: If the intersection of any two columns of the equivalent binary matrix *D* have at most one nonzero element, then C_1 is in its minimum form.

Our aim in solving the ambiguity problem is to first identify ambiguities, and subsequently to describe them in the simplest possible way that corresponds to a minimum form of the linear combination matrix C_1. Useful results that are closely related to Lemma 3 describe the existence of surely testable components, canonical ambiguity groups and ambiguity clusters.

Lemma 4: A circuit component is surely testable if and only if the corresponding row of

C_1 is zero.

In order to define the canonical ambiguity groups and the ambiguity clusters, let us identify a set of elements of the co-basis $a_2 = \{a_{21}a_{22}...a_{2k}\}$ that corresponds to a union of columns of *D* such that for each column that corresponds to $a_{2j} \in a_2$, there exists another column that corresponds to $a_{2i} \in a_2$, $a_{2j} \neq a_{2i}$ such that the two columns have a nonempty intersection. This set of columns can be easily obtained from the matrix *D* using less than $O((p-r)^3)$ operations. Let the set of the elements of the basis $a_1 = \{a_{11}a_{12}...a_{1k}\}$ corresponds to nonzero rows in the set of columns described by a_2.

Lemma 5: a set of components described by the union $a = a_1 \cup a_2$ constitutes an ambiguity group of the testability matrix B.

Ambiguity groups identified by Lemma 5 are either ambiguity clusters or canonical ambiguity groups. The following theorem can identify canonical ambiguity groups.

Lemma 6: the ambiguity group represented by the set $a = a_1 \cup a_2$ is canonical if and only if cardinality of a_2 is equal to one.

As a result of a single QR run we were able to identify all canonical ambiguity groups and all surely testable components in the testability matrix. The remaining task is to analyze the ambiguity clusters.
Let us assume that $a = a_1 \cup a_2$ is an ambiguity cluster and select $A \subset B$ as a subset of columns of *B* which corresponds to the ambiguity cluster a. We define a minimum form partition of an ambiguity cluster as a minimum form partition of the selected submatrix *A* of the testability matrix. If the QR factorization is repeated on the submatrix *A* and a_1 columns are selected for the basis b_1 with columns a_2 selected as the co-basis b_2, then the resulting equivalent binary matrix D_a will be a submatrix of *D* obtained on the intersection of rows that correspond to a_1 and partition of the cluster elements between elements of the basis and the co-basis. At this point any further reduction in the number of nonzero elements of D_a will result from swapping an element of the basis with an element of the co-basis.

2.4 Pseudo Solution matrix

Using a pseudo inverse of *B* we can determine a solution of (1) as follows

$$\overline{P} = pinv(B)\ M \qquad (6)$$

Depending on the structure of C_1 we may have or may not have determined correct values of the faulty parameters. From (1) we can identify a partition of the solution components into P_1 and P_2 as follows:

$$BP = B_1[IC_1]\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = M \tag{7}$$

where P_1 corresponds to independent columns of B in a selected minimum form of C_1.

3. Fault Identification Techniques In Low Testability Systems

3.1 Minimum Form Solution

Since all ambiguity groups involve different columns of B_1 matrix, their solutions can be obtained independently of each other. Let us consider a partition of (1) according to the ambiguity groups:

$$BP = \sum_{i=1}^{g} B_i P_i = \sum_{i=1}^{g} M_i = M \tag{8}$$

where B_i contains columns of B, which correspond to i-th ambiguity group (including independent columns of B). We know that in a low testability system the solution vector P is not unique, however, vectors M_i in equation (8) do not depend on a specific solution vector. In particular, M_i for each ambiguity group (including independent components) can be uniquely calculated using $\overline{P}$. Solving (1) with minimum number of faulty parameters will be obtained by solving each ambiguity group separately. Therefore, our aim is to solve

$$B_i P_i = M_i \qquad \text{i} = 1,\dots,\text{g} \tag{9}$$

with a minimum number of nonzero components in P_i. We call such solution a **minimum form solution**.

3.2 Uniquely Identified Parameter

If a parameter does not belong to any ambiguity group, then this parameter can be uniquely identified according to Lemma 1. A minimum number of faulty parameters that can satisfy test equations should be identified.

3.3 Equivalent Fault Vector

Using the minimum form partition for i-th ambiguity group we obtain

$$B_i P_i = B_{1i}[IC_{1i}]\begin{bmatrix} P_{1i} \\ P_{2i} \end{bmatrix} = B_{1i}\hat{P} = M_i \tag{10}$$

where

$$\hat{P}_i = P_{1i} + C_{1i}P_{2i} \tag{11}$$

is an equivalent set of parameter deviations in i-th ambiguity group and is called the

equivalent fault vector. Since matrix B_{1i} has the full column rank, the equivalent fault vector is unique. However, in general, a solution for P_{1i} *and* P_{2i} which satisfy (11) is not unique. Since our aim is to solve test equations with a minimum number of faulty components, we can obtain $\hat{P}_i$ by forcing all P_{2i} to zero, therefore the largest number of faults we will consider equals to the size of P_{1i} which is equal to the rank of the ambiguity group. Notice, that a single faulty parameter $p \in P_{2i}$ may generate a number of equivalent faults (nonzero values) in $\hat{P}_i$. On the other hand, if a parameter $\hat{p} \in \hat{P}_i$ is zero, then the corresponding parameters of P_i are zero with probability equal to one as stated in the following lemma.

Lemma 7: if an element $\hat{p} \in \hat{P}_i$ of the equivalent fault vector obtained by solving (10) equals zero, with $\hat{p} = p_{1k} + \sum_j c_{kj} p_{2j}$, where $p_{1k} \in P_{1i}$, and $p_{2j} \in P_{2i}$, then $p_{1k} = p_{2j} = 0$ with probability equal to one (excluding a singular subspace in the solution space).

3.4 Solution Invariant Matrix and Proper Cofactor

Let us define the **solution invariant** matrix for *i*-th ambiguity group as a concatenation of the equivalent fault vector and the linear combination matrix of this ambiguity group as follows:

$$S_i = [\ \hat{P}_i\ C_{1i}] \tag{12}$$

Even if Lemma 8 is not satisfied, a number of parameter deviations can be set to zero depending on the cofactors of the solution invariant matrix. Let us consider a cofactor of S_i, which contains as its first column elements of $\hat{P}_i$. We call it a **proper cofactor** if its remaining columns are linearly independent. Since a proper cofactor must include at least one element of $\hat{P}_i$, its minimum size is one. In addition, it cannot have zero columns except the first one. If a proper cofactor is zero, then $\hat{P}_i$ can be expressed by smaller number of nonzero parameters P_{1i} *and* P_{2i} than its size, which means that the number of nonzero parameters in the ambiguity group is less than its rank. We call such proper cofactor a **nullifying cofactor**.

In the procedure of finding a minimum form solution we will identify nullifying cofactors, set selected parameters to zero, and modify the solution invariant matrix S_i. Let us denote by $\boldsymbol{a_1}$ a set of the basis elements that corresponds to rows of C_{1i}, and by $\boldsymbol{a_2}$ a set of the cobasis elements which correspond to columns of C_{1i}. A nullifying cofactor is obtained on the intersection of rows $a_r \subseteq a_1$ and columns $a_c \subseteq a_2$ of matrix C_{1i} and is represented by its submatrix $\boldsymbol{C_{rc}}$. A subset of columns $a_n \subseteq a_2 - a_c$ which has nonempty intersection with rows $\boldsymbol{a_r}$ in the C_{1i} matrix is called **nullifying columns.**

Lemma 8: if none of nullifying cofactors of solution invariant matrix S_i exists, and for minimum form of C_1, which is related to the equivalent fault vector $\hat{P}$ by equation $\hat{P} = P_1 + C_1 P_2$, there is the *m*th row of C_1 that has only one nonzero element in its *n*th column, then $(P_1)_m = 0$, and $(P_2)_n = \hat{P}_m / (C_1)_{mn}$. Under this situation, $\hat{P}_m = (P_1)_m + (C_1)_{mn} (P_2)_n$.

In order to find minimum form solution, there are two possible solutions, either $(P_1)_m = 0$ or $(P_2)_n = 0$. Since none of nullifying cofactors of solution invariant matrix exists, if $(P_2)_n$ is set to zero, there will not be other chances to find nullifying cofactor of matrix S_i after the effects of both $(P_1)_m$ and $(P_2)_n$ are deducted from the equivalent vector $\hat{P}$. So the results should be $(P_1)_m = 0$, and $(P_2)_n = \hat{P}_m / (C_1)_{mn}$.

3.5 Reduction of Degrees of Freedom in the Solution Vector:

The following procedure is performed sequentially for each nullifying cofactor to reduce the degrees of freedom in the solution vector:

Step 1. Find a nullifying cofactor of S_i. If it exist, go to step 2. If none exists, check each row of the minimum form of C_1 that satisfies the equation $\hat{P} = P_1 + C_1 P_2$. If in the mth row of C_1, only the element in the nth column is not zero, then $\hat{P}_m = (P_1)_m + (C_1)_{mn} (P_2)_n$. According to Lemma 9, $(P_1)_m = 0$, and $(P_2)_n = \hat{P}_m / (C_1)_{mn}$. Next, jump to step 5. If both situations are not satisfied, then stop. In other general cases, if $(n \times k)$ solution invariant matrix has no nullifying cofactors, then it yields $\begin{vmatrix} n+k-1 \\ n \end{vmatrix}$ different solutions of the test equation.

Step 2. Set all parameters of P_{1i} which correspond to $\boldsymbol{a_r}$ to zero.

Step 3. Set all parameters of P_{2i} which correspond to $\boldsymbol{a_n}$ to zero.

Step 4. Solve $\hat{P}_r = C_{rc} P_{2c}$, where $\hat{P}_r \subseteq \hat{P}_i$ corresponds to rows r. The number of elements of P_{2c} is smaller by one than the number of elements of $\hat{P}_r$. P_{2c} is a subset of faulty parameters.

Step 5. Subtract $C_{1c} P_{2c}$ from $\hat{P}_i$ and remove rows $\boldsymbol{a_r}$ to define a new value of $\hat{P}_i$ for the next iteration of this procedure, where C_{1c} is composed of c columns of C_{1i}.

Step 6. Remove rows $\boldsymbol{a_r}$ and columns $a_c \cup a_n$ from matrix C_{1i} and subsequently S_i.

Step 7. Go back to 1.

Each nullifying cofactor reduces the number of nonzero parameters in the solution of low testability system by one. Since our aim is to find a minimum form solution, we would like to find many (usually small size) nullifying cofactors. In addition, since the removal of nullifying columns reduces the size of S_i, in order to be able to find more nullifying cofactors a cofactor with small number of nullifying columns is preferred.

4. Computer Simulation

A C++ program was developed to extract system admittance matrix from SPICE format circuit input file. This contains modeling of complex components in mixed mode system, such as CMOS transistor, NPN transistor, motor and so on. The C++ program provides output files including information about the parameter value and location. These output files provide interface to the Matlab file, which will further realize the sensitivity analysis algorithm.

In addition, a Matlab program was developed to find minimum form solution for low testability systems. First, for system test equation $BP = M$, it used the function MINSOLGROUP to get the minimum form of the ambiguity group. At the same time the corresponding base vectors of matrix B was found to obtain matrix B_1. Since $B = B_1\left[I \quad C_1\right]$, the minimum form of C_1 could be extracted from $pinv(B_1)\times B$, where $pinv(B_1)$ was the pseudoinverse of matrix B_1. Then it got equivalent fault vectors $\hat{P} = pinv(B_1)\times M$. Next, it constructed solution invariant matrix $S_i = \left[\hat{P}_i \quad C_{1i}\right]$. At last, the sequential steps described in Procedure 3 previously were followed to try to find nullifying cofactor, so that the degrees of freedom in the solution vector could possibly be deducted until it found the final minimum form solution.

The block diagram of matlab program is illustrated in Fig. 1.

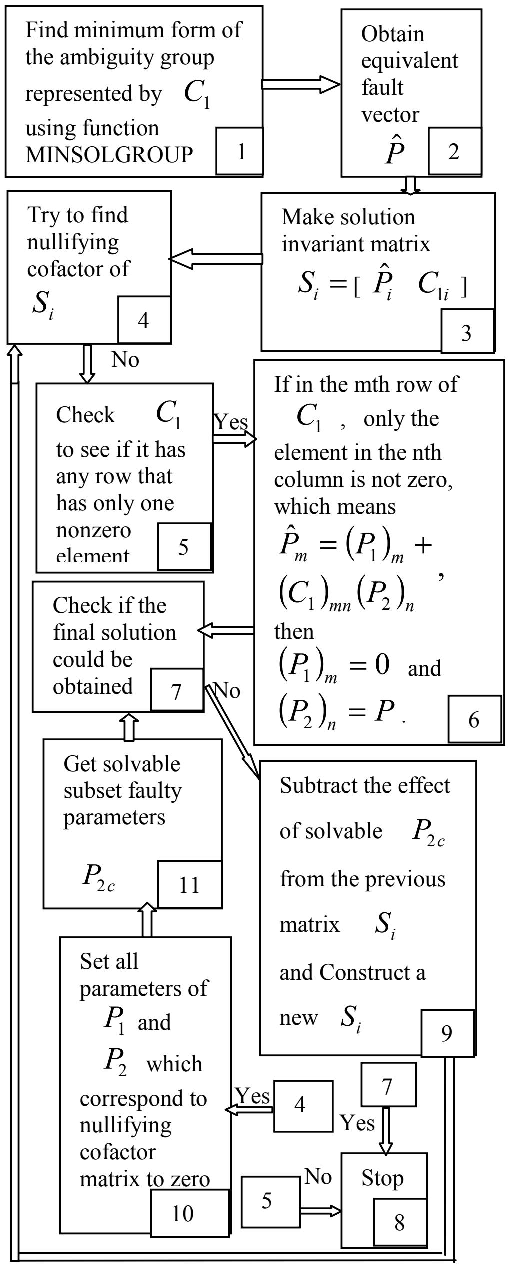

Fig.1. Block Organization of the program MINSOLGROUP

These programs provide the efficient methods to solve low testability system.

5. Experimental Results

Example 1: As an example, let us solve the following test equation:

$$BP = \begin{bmatrix} 2 & 1 & 3 & 5 & 8.1 & 1 \\ 8 & 4 & 8 & 20 & 24.4 & 4 \\ 4 & 9 & 1 & 31 & 6.9 & 9 \\ 6 & 9 & 1 & 33 & 8.9 & 9 \\ 7 & 4 & 2 & 19 & 11.4 & 4 \\ 9 & 8 & 2 & 33 & 13.8 & 8 \\ 7 & 6 & 6 & 25 & 19.6 & 6 \\ 8 & 7 & 3 & 29 & 14.7 & 7 \\ 2 & 1 & 5 & 5 & 12.1 & 1 \\ 4 & 2 & 7 & 10 & 18.2 & 2 \\ 1 & 3 & 5 & 10 & 11.3 & 3 \\ 4 & 2 & 3 & 10 & 10.2 & 2 \end{bmatrix} \begin{bmatrix} 1.6 \\ 0 \\ 0 \\ 1.8 \\ 0 \\ 0 \end{bmatrix} = M = \begin{bmatrix} 12.2 \\ 48.8 \\ 62.2 \\ 69 \\ 45.4 \\ 73.8 \\ 56.2 \\ 65 \\ 12.2 \\ 24.4 \\ 19.6 \\ 24.4 \end{bmatrix} \tag{13}$$

Notice that there are two faults $P_1 = 1.6$, and $P_4 = 1.8$. Suppose that we know *B* and *M*, and parameters *P* are unknown. Using the pseudo inverse (6) we can find

$$\overline{P} = \begin{bmatrix} -1.3988 \\ 0.1069 \\ -0.5638 \\ 1.7194 \\ 0.2819 \\ 0.1069 \end{bmatrix} \tag{14}$$

As we can see faulty elements are not correctly identified. By using the program to identify minimum form of the ambiguity groups we can get

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 0.1 & 1 \\ 0 & 2 & 0 \end{bmatrix} \tag{15}$$

which indicates that there is only one ambiguity group which contains all parameters, therefore only one equation (9) must be considered. Using partition for this ambiguity group we get

$$B_i P_i = B_{1i} \left[I C_{1i} \right] \begin{bmatrix} P_{1i} \\ P_{2i} \end{bmatrix} = B_{1i} \hat{P}_i$$

$$B_{1i}\hat{P}_i = \begin{bmatrix} 2 & 1 & 3 \\ 8 & 4 & 8 \\ 4 & 9 & 1 \\ 6 & 9 & 1 \\ 7 & 4 & 2 \\ 9 & 8 & 2 \\ 7 & 6 & 6 \\ 8 & 7 & 3 \\ 2 & 1 & 5 \\ 4 & 2 & 7 \\ 1 & 3 & 5 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3.4 \\ 5.4 \\ 0 \end{bmatrix} \tag{16}$$

Since $\hat{p}_3 = 0, \hat{p}_3 \in \hat{P}_i, and\ \hat{p}_3 = p_{13} + 2 * p_{22}$, therefore, based on Lemma 2, parameters $p_{13} = p_{22} = 0$, which means that the original parameters $p_3 = p_5 = 0$. The solution invariant matrix is as follows:

$$s_i = \begin{bmatrix} \hat{P}_i & C_{1i} \end{bmatrix} = \begin{bmatrix} 3.4 & 1 & 1 & 0 \\ 5.4 & 3 & 0.1 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix} \tag{17}$$

Now, applying Procedure 3 we identify one nullifying cofactor on intersection of rows $a_r = \{3\}$ and columns $a_c = \{\varnothing\}$ of matrix $C_{1i.}$ The nullifying columns are those columns of $a_2 - a_c = a_2$ which have nonempty intersections with rows $a_{r,}$ so $a_n = \{2\}$. According to Procedure 3 we set the parameters p_{13} and p_{22} to zero. Procedure 3 continues after removing row 3 and column 2 of C_{1i} to obtain a reduced matrix S_i.

$$S_i = \begin{bmatrix} 3.4 & 1 & 1 \\ 5.4 & 3 & 0.1 \end{bmatrix} \tag{18}$$

Since this matrix does not have a nullifying cofactor the procedure 3 is finished. The remaining parameters are chosen to satisfy the resulting solution invariant matrix. In this case we solve the following equation (equivalent to (11))

$$\hat{P}_i = P_{1i} + C_{1i}P_{2i} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 0.1 \end{bmatrix} \begin{bmatrix} P_4 \\ P_6 \end{bmatrix} \tag{19}$$

Since we are seeking a minimum form solution, we can get six different solutions of equation (16) as follows:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 3.4 \\ 5.4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 0 \\ 1.8 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 3.4 \\ 0 \\ 0 \\ 5.4 \end{bmatrix}, \begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.06 \\ 0 \\ 3.4 \end{bmatrix},$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.8 \\ 3.4 \\ 0 \end{bmatrix}, \begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3.4 \\ -4.8 \end{bmatrix} \tag{20}$$

These four vectors combined with $p_3 = p_5 = 0$ represent the minimum form solutions of the test equation considered.

Example 1 illustrated the case in which there were several minimum form solutions. This was the result of the solution invariant matrix without nullifying cofactors. If the nullifying cofactors remove all the elements of the solution invariant matrix, then the minimum form solution of test equations is unique as is illustrated in the following example.

Example 2: Minimum form solution of test equations with unique identified parameters.

Consider the following test equation:

$$M = BP = \begin{bmatrix} 2 & 1 & 3 & 11 & 8.1 & 1 \\ 8 & 4 & 8 & 36 & 24.4 & 4 \\ 4 & 9 & 1 & 33 & 6.9 & 9 \\ 6 & 9 & 1 & 35 & 8.9 & 9 \\ 7 & 4 & 2 & 23 & 11.4 & 4 \\ 9 & 8 & 2 & 37 & 13.8 & 8 \\ 7 & 6 & 6 & 37 & 19.6 & 6 \\ 8 & 7 & 3 & 35 & 14.7 & 7 \\ 2 & 1 & 5 & 15 & 12.1 & 1 \\ 4 & 2 & 7 & 24 & 18.2 & 2 \\ 1 & 3 & 5 & 20 & 11.3 & 3 \\ 4 & 2 & 3 & 16 & 10.2 & 2 \end{bmatrix} \begin{bmatrix} 1.6 \\ 0 \\ 0 \\ 1.8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 23 \\ 77.6 \\ 65.8 \\ 72.6 \\ 52.6 \\ 81 \\ 77.8 \\ 75.8 \\ 30.2 \\ 49.6 \\ 37.6 \\ 35.2 \end{bmatrix} \tag{21}$$

Which is a slightly modified equation (10) with the same faults inserted. The pseudo inverse (6) obtained in this case equals to $\overline{P}$.

$$\overline{P} = \begin{bmatrix} 1.3704 \\ 0.3633 \\ -0.4593 \\ 1.5416 \\ 0.4881 \\ 0.3633 \end{bmatrix} \tag{22}$$

The minimum form of the ambiguity group is as follows:

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 0.1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \tag{23}$$

and the solution vector (11) of equation (10) equals to

$$\hat{P}_i = \begin{bmatrix} 3.4 \\ 5.4 \\ 3.6 \end{bmatrix} \tag{24}$$

The solution invariant matrix

$$S_i = \begin{bmatrix} \hat{P}_i & C_{1i} \end{bmatrix} = \begin{bmatrix} 3.4 & 1 & 1 & 0 \\ 5.4 & 3 & 0.1 & 1 \\ 3.6 & 2 & 2 & 0 \end{bmatrix} \tag{25}$$

is analyzed according to Procedure 3.
Step 1. The nullifying cofactor

$$N_i = \begin{bmatrix} 5.4 & 3 \\ 3.6 & 2 \end{bmatrix} \tag{26}$$

is identified with $a_r = \{2\ 3\}, \quad a_c = \{1\}, \quad and \quad a_n = \{2\ 3\}$.

Step 2. Parameters p_{12} and p_{13} are set to zero.

Step 3. Parameters p_{22} and p_{23} are set to zero.

Step 4. Solve the following equation

$$\hat{P}_r = \begin{bmatrix} 5.4 \\ 3.6 \end{bmatrix},$$
$$C_{rc}P_{2c} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} P_{21} \tag{27}$$

to identify the first faulty parameter p_{21}=1.8.

Step 5. Subtract the following equation from $\hat{P}_i$

$$C_{1c}P_{2c} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \bullet 1.8 = \begin{bmatrix} 1.8 \\ 5.4 \\ 3.6 \end{bmatrix} \tag{28}$$

and remove rows $\boldsymbol{a_r}$ to obtain a new value of $\hat{P}_i = [1.6]$.

Step 6. Remove rows $\boldsymbol{a_r}$ and columns $a_c \cup a_n$ from matrix C_{1i} and subsequently S_i. The resulting new matrix S_i=[1.6] corresponds to the second faulty parameter p_{11}=1.6. In this case both faulty parameters were uniquely identified.

If models for complex components can be developed properly, these methods can be extended to many mixed mode low testability system. In this paper, one example of DC motor is analyzed, which combines electronic circuits with mechanical components forming a simple electro-mechanical system.

Example 3: Open loop DC motor

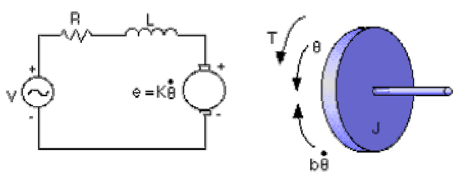

Fig. 2. Electronic circuit with open loop DC motor

This system can be described by the equation:

$$\frac{d}{dt}\begin{bmatrix} d\theta / dt \\ i \end{bmatrix} = D \times \begin{bmatrix} d\theta / dt \\ i \end{bmatrix} + C \times V \tag{29}$$

where $D = \begin{bmatrix} 1/J & K/J \\ -K/L & R/L \end{bmatrix}$, and $C = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$. *J* is moment of inertia of the rotor, *b* is the damping ratio of the mechanical system, and *K* is the electromotive force constant. Here, we use *J*=0.01 $kg \cdot m^2 / s^2$, *b*=0.1Nms, *K*=0.01Nm/Amp, *R*=1ohm, *L*=0.5H, *V*=1v. For simulation purpose, we set one fault *Δb*=0.001. The system test equation is constructed in the following way:

$$\begin{bmatrix} d\theta/dt_{n+1} \\ i_{n+1} \end{bmatrix} = D \times \begin{bmatrix} d\theta/dt_n \\ i_n \end{bmatrix} \times \Delta t + C \times V \times \Delta t + \begin{bmatrix} d\theta/dt_n \\ i_n \end{bmatrix} \tag{30}$$

where $d\theta/dt_o = 0$,and $i_0 = 0$.

$$\begin{aligned} \begin{bmatrix} d(d\theta/dt)_{n+1}/dh \\ di_{n+1}/dh \end{bmatrix} &= D \times \begin{bmatrix} d(d\theta/dt)_n/dh \\ di_n/dh \end{bmatrix} \times \Delta t \\ &+ dC/dh \times V \times \Delta t + dD/dh \times \begin{bmatrix} d\theta/dt_n \\ i_n \end{bmatrix} \times \Delta t \\ &+ \begin{bmatrix} d(d\theta/dt)_n/dh \\ di_n/dh \end{bmatrix} \end{aligned} \tag{31}$$

where h is a group of parameters $(b \quad K \quad J \quad R \quad L)$. We can calculate di_n/dh in the similar way for each integer n at the same time interval. Moreover, we can get the measurement matrix *M* by deducting i, $d\theta/dt$ from i_{error} and $d\theta_{error}/dt$. As a result, we find only one ambiguity group $(\Delta b \ \Delta K \ \Delta J \ \Delta R \ \Delta L)$. Then, using our fault diagnosis method, we can finally find fault *Δb*=0.001 successfully.

6. CONCLUSIONS

In this paper, we focus on a solution method for low testability analog systems. Although a unique solution is not always possible in such systems, minimum form solution is possible under the assumption that the number of faulty parameters in VLSI circuits, which have to be identified using the test equations, is small. The minimum form of the ambiguity groups has to be found first. For each ambiguity group, the linear combination matrix can be obtained so that the equivalent fault vector can be built. The concept of the equivalent fault vector is extremely useful, because although the system test equations usually have various solutions, the solution for the equivalent fault vector is unique according to the Lemma 7 we described in this paper. Then it is possible to construct the solution invariant matrix. The solution procedure starts from this point to try to reduce the degrees of freedom in the solution vector. From now on, new concepts such as equivalent fault vector, solution invariant matrix, proper cofactor, and nullifying cofactor are introduced to help understand and solve the problem. Moreover, different approaches described in Lemma 6, Lemma 7 and Lemma 8 are combined in the solution procedures, and the same solving methods are repeated for each renewed invariant matrix until the final solutions are found.

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