Entropy-Based Optimum Test Points Selection for Analog Fault Dictionary Techniques

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***Abstract*—An efficient method to select an optimum set of test points for dictionary techniques in analog fault diagnosis is proposed. This is done by searching for the minimum of the entropy index based on the available test points. First, the two-dimensional integer-coded dictionary is constructed whose entries are measurements associated with faults and test points. The problem of optimum test points selection is, thus, transformed to the selection of the columns that isolate the rows of the dictionary. Then, the likelihood for a column to be chosen based on the size of its ambiguity set is evaluated using the minimum entropy index of test points. Finally, the test point with the minimum entropy index is selected to construct the optimum set of test points. The proposed entropy-based method to select a local minimum set of test points is polynomial bounded in computational cost. The comparison between the proposed method and other reported test points selection methods is carried out by statistical experiments. The results indicate that the proposed method more efficiently and more accurately finds the locally optimum set of test points and is practical for large scale analog systems.**

***Index Terms*—Analog fault diagnosis, fault dictionary, rough set, test point.**

I. INTRODUCTION

TESTABILITY of analog circuits has gained more attention recently due to the rapid development in analog VLSI chips, mixed-signal systems, and system-on-chip (SoC) products. It is usually classified into two main categories [1], [2]: simulation-before-test (SBT), including probabilistic and fault dictionary techniques, and simulation-after-test (SAT), including optimization, fault verification, and parameter identification techniques.

For analog systems with mostly catastrophic faults, fault dictionary techniques are popular choice [3]. A fault dictionary is a set of measurements of the circuit-under-test (CUT) simulated under potentially faulty conditions (including fault-free case) and organized before the test. The measurements could be at different test nodes, test frequencies, and sampling times. In this paper, all of them are defined as the **test points**. To construct a fault dictionary, all potential faults are listed and the stimuli type (dc, ac, or time domain), shape, frequency, and amplitude are selected. The CUT is then simulated for the fault-free case and all faulty cases. The signatures of the responses are stored and organized in the dictionary for use in the on-line fault diagnosis. Before testing, the optimum selection of test points is required to achieve the desired degree of fault diagnosis and to maintain a reasonable computational cost. At the testing stage, the same stimuli as those used in constructing the dictionary are applied to the faulty CUT. The measurement signatures at selected test points are compared with those prestored in the dictionary to match the fault(s) to one of predefined faults or to a set of faults according to the preset criteria.

Fault dictionary techniques are usually used to diagnose the open or short faults [3] with possible inclusion of the parametric fault diagnosis [4], [5]. Fault dictionary techniques have the advantage of minimum on-line computation, but a significant off-line computation needed to construct the database limits their application. Optimum selection of test points is, therefore, important to reduce the computation cost by reducing the dimensionality of the fault dictionary. Simultaneously, optimum selection of test points reduces the test cost by eliminating redundant measurements. The emphasis of this paper is on the selection of an optimum set of test points. One straightforward solution is to have an exhaustive search for such a set with minimum size to fully isolate the faults. As discussed in Section III, the exhaustive search is proven to be NP-hard. Therefore, it is not practical, considering the computation cost, while any polynomial-bounded method cannot guarantee such a global minimum set. The tradeoff between the desired degree of fault diagnosis and computation cost is to select a local minimum set.

A heuristic method for test points selection based on the concept of confidence levels was proposed by Varghese *et al.* [6] with an expensive complexity of $O(kfn(f+n))$ [7] where f is the number of faults, n is the total number of system nodes, and k is the number of times to compute the confidence level. Stenbakken and Souders [8] proposed a method using the QR factorization of a system sensitivity matrix. The complexity is primarily determined by the complexity of QR factorization $O(n^3)$. Abderrahman *et al.* [9] used sequential quadratic programming and constraint logic programming to generate test sets. Lin and Elcherif [3] proposed two heuristic methods based on two criteria proposed by Hochward and Bastian [10]. The complexities are $O(f^2p^2)$ [13] and $O(f^2p)$ [7], respectively, where p is the number of examined test nodes. Spaandonk and Kevenaar [11] looked for a set of test points by combining the decomposition method of system sensitivity matrix and an iterative algorithm. A set of test points whose size is equal to the rank of system sensitivity matrix is selected randomly. Then, in the iterative algorithm, they randomly exchange a test point in the set with a randomly selected integer in order to compute D, the determinant of covariance matrix. The new set is accepted if it has a lower D than the previous set. Manetti *et al.* [12] wrote

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a computer program by PROLOG to automatically select a set of test points for linear analog systems. The program was based on a knowledge base constituted by some simple rules derived from experience and heuristic reasoning. It was an example application of expert system. All these methods are to find a local minimum set with complexity which is polynomial bounded. Prasad and Pinjala [7] proposed a polynomial-bounded method with complexity of $O(fp)$, but the selection of hashing function needed is difficult for general test cases. Prasad and Babu [13] proposed four algorithms based on three strategies for inclusive approaches and three strategies for exclusive approaches. The complexity is $O(fp\log f)$ or $O(f(p+m)logf)$ where m is the number of the final selected test points, hence it is also polynomial bounded. Note that the so called "minimal set" in [13] is a local minimum set.

Test points selection techniques can be classified into two categories: inclusive and exclusive [13]. For the inclusive approaches, the desired optimum set of test points is initialized to be null, then a new test point is added to it if needed. For the exclusive approach, the desired optimum set is initialized to include all available test points. Then a test point will be deleted if its exclusion does not degrade the degree of fault diagnosis.

In this paper, a polynomial-bounded method to select a locally minimum set of test points is proposed and compared with the other reported methods. In Section II, an integer-coded dictionary is constructed and its relation with rough set theory is summarized. Then, the proposed method is described in Section III to isolate the dictionary rows with selected columns by searching for the minimum entropy index of test points. Section IV demonstrates the superiority in the computational efficiency and solution quality of the proposed entropy-based method by comparing it with the other reported methods based on statistical computer simulations. Finally, brief conclusions are given in Section V.

II. INTEGER-CODED DICTIONARY METHODS IN VIEW OF ROUGH SET THEORY

A. Integer-Coded Dictionary

There is an important phenomenon which is commonly encountered and difficult to solve in analog testing and fault diagnosis: measurement ambiguity. Distinct faults may result in the measurements whose values are close to each other. Therefore, it is difficult to clearly recognize the specific fault. Such faults are said to be in the same ambiguity set associated with a specific measurement. The concept of an **ambiguity set** was first introduced by Hochwald and Bastian [10]. It is defined as a list of faults which fall in a distinguished band of measurement levels, which could be determined by Monte Carlo simulation considering component tolerances, tester errors, and the optimum partition methods. For the optimum test points set to distinguish the ambiguity sets, the integer-coded dictionary was first proposed by Lin and Elcherif [3] and subsequently researched by Prasad and Babu [13]. This approach proved to be an effective tool for the optimum test points selection.

The two-dimensional integer-coded dictionary [3], or fault-wise table [13], is constructed as follows. Its rows represent all the potential faults (including fault-free case) while its columns represent all the available test points. For each test point, different ambiguity sets are classified based on computer simulation according to a preset criterion. A specific integer code is then assigned to each ambiguity set. Thus, the entries of the dictionary correspond to the simulated system responses. Note that for a given test point, distinct ambiguity sets have distinct integer codes. However, the same integer code can be assigned to different ambiguity sets associated with different test points, because each test point is an independent measurement and ambiguity sets for the same test points are independent.

Let $F = \{f_0, f_1, \ldots, f_f\}$ be the set of all potential faults to be diagnosed (including the fault-free case f_0) and $N = \{n_1, n_2, \ldots, n_t\}$ be the set of all available test points where subscript f is the number of potential faults and t is the number of available test points. To define the concept of diagnosable circuit, let us associate the measurement equivalent classes which are the elements of the dictionary A as follows:

$$\forall f_i \in F(0 \le i \le f) \text{ and } \forall n_j \in N(1 \le j \le t),$$
$$\exists \phi(f_i, n_j) = a_{ij} \in A.$$

where a_{ij} is element of the dictionary A corresponding to the ith fault and jth test point.

Definition 1: Let $A_j = \{a_{0j}, a_{1j}, a_{2j}, \ldots, a_{fj}\} \subset A$ *be a subset of dictionary* A *with a single test point* n_j*. If for any pair of elements (*$a_{pj} \in A_j, a_{qj} \in A_j, p \neq q$*), we have* $a_{pj} \neq a_{qj}$*, then the CUT is* ***diagnosable*** *by test point* n_j*.*

It would be optimistic to expect the CUT to become diagnosable by using a single test point n_j. Usually, we could have different faults f_p and $f_q (p \neq q)$ with identical dictionary elements $a_{pj} = a_{qj} = i$, where i is the integer code. Under such conditions, we claim that faults f_p and f_q belong to an ambiguity set S_i^j associated with test point n_j and integer code i where ambiguity set S_i^j is defined as follows.

Definition 2:

$$S_i^j = \{f_m \in F \,| a_{mj} = i, 0 \le m \le f\}.$$

An immediate proposition as a result of Definitions 1 and 2 is obtained.

Proposition 1: The CUT corresponding to the dictionary A *is* ***diagnosable*** *for a set of test points* $N_i \subset N$ *if and only if for any pair of faults* $\{f_p \in F, f_q \in F, p \neq q\}$*, there always exists* $n_j \in N_i$ *with* $a_{pj} \neq a_{qj}$*.*

The integer-coded dictionary provides information about the ambiguity sets for each test point. In view of analog fault diagnosis, the purpose of test points selection is to isolate the faults by the minimum number of test points. In view of integer-coded dictionary, the purpose is to distinguish rows by the minimum number of columns. Hence, the integer-coded dictionary transformed the problem of test points selection into the selection of dictionary columns to isolate the rows.

B. Application of Rough Set Theory in Test Points Selection

As pointed out in Section I, one straightforward method of test points selection is to have an exhaustive search. The obtained global minimum set will help to check the solution accuracy of any polynomial-bounded method which only guarantees the local minimum set. Searching for a global minimum can be implemented by algorithms in rough set theory. As an efficient mathematical tool to represent the knowledge that can be extracted from the database, rough set theory has widespread applications from data mining and decision support to artificial intelligence (AI), information systems, pattern recognition, and neural networks. Fault dictionary techniques, or, speaking precisely, optimum test points selection of these techniques are based on pattern recognition methods [2]. Conceivably, a number of available techniques in pattern recognition will find their applications in test points selection. Rough set theory is not an exception. For the purpose of optimum test points selection, the features of rough set theory, such as knowledge representation, knowledge reduction, and imprecise knowledge capturing, are especially useful.

In order to make use of the rough set algorithms in test points selection, some well-recognized concepts in rough set are defined with equivalent concepts in testing. In view of rough set theory, an ambiguity set corresponds to the equivalence class [14] or the indiscernible set [15]. The ambiguity reduction of the fault dictionary is equivalent to the computation of reducts and cores [14], [15] and the optimum test points selection corresponds to simplification of the decision table [14]. Inclusive approaches to test points selection produce discerns [16] and exclusive approaches produce reducts [14], [15], which can be explained as the set of test points to diagnose the faults in the dictionary. The concept of core [14], [15] can be explained in view of dictionary as the indispensable measurements to separate the faults.

The integer-coded dictionary is a special case of the decision table [14], [15]. The faults in the dictionary correspond to the decision attributes in the decision table, while the test points correspond to the condition attributes. The faults have to be uniquely separated so that we can explain the faults as the decision attributes, which are essentially integer-coded equivalence classes $D = (0, 1, \ldots, n)^T$ and are implicit in the dictionary. The superscript T denotes transpose of vector. Hence, the integer-coded dictionary is a special case of the decision table with one column matrix D where no two entries are the same. Consequently, a number of algorithms developed for the simplification of decision table and computation of reducts and cores in the rough set theory could be applied for the optimum selection of test points.

The concepts in the optimum selection of test points and their analogues in rough set theory are listed in Table I.

TABLE I
TERMINOLOGY USED IN OPTIMUM TEST POINTS SELECTION AND ROUGH SET THEORY

Concepts in optimum selection	Concepts in rough set theory
integer coded dictionary, or fault-wise table	decision table
inclusive approaches	algorithms of finding discerns
exclusive approaches	algorithms of finding reducts
test points set	reduct
ambiguity set	equivalence class, or indiscernible set
selection of optimum test points	computation of cores and reducts

Based on the rough set theory, searching for the global minimum set of test points can be implemented as the following procedure, which falls into exclusive category. The first thing is to guarantee that there is no ambiguity in the dictionary A. If ambiguities occur, one or both rows must be removed. Then, the core of the dictionary A is determined. Try to remove one column in the dictionary. If ambiguities occur after removal, the removed column is concluded to be a part of the core. Try one column at a time until all columns have been checked. The core is obtained by the set of all columns which produced ambiguities. These measurement columns must be included in each reduct. Finally, reducts are computed. The core is first checked to see if it is a reduct. If it is, the core is concluded as the minimum reduct. Otherwise, add all possible combinations of the remaining test points to the core set and check for ambiguities. The set of test points with minimum size without producing ambiguities is the desired reduct with minimum size.

A computer program EXPANSION written with MATLAB is based on the above procedure to search for the global minimum set [16]. The above proposed rough set-based method was shown to be NP-hard in [17] and, therefore, is computationally expensive. The unique aim of the above rough set-based procedure is to find the exact size of the minimum test set for the purpose of reference for the proposed entropy-based method and other test points selection methods.

III. ENTROPY-BASED OPTIMUM TEST POINT SELECTION

It can be shown that the computation time to search for the global minimum set of test points is not polynomial bounded [3]. The search for a global minimum set can be implemented by polynomial-bounded methods, but the global minimum cannot be guaranteed. In view of a rough set theory, the search for a global minimum corresponds to searching for a minimum size reduct [14]. It was proven in [17] that finding a minimum size reduct is NP-hard; hence, there is no polynomial-bounded algorithm which can guarantee the minimum set of test points.

To efficiently achieve the desired degree of fault diagnosis, an alternative solution is to search for a local minimum set of test points. In this paper, an entropy index computation method, which belongs to inclusive approaches, is proposed to efficiently search for a test points set. The idea is similar to the work by Hartmann *et al.* [18] in which an entropy-based method was developed to efficiently construct decision tree. The method developed in [18] deals only with attributes of Boolean logic and is extended in our method to deal with attributes of integer codes.

The dominant idea of the proposed method is to evaluate the probability for each fault to be separated in accordance with the cardinality of each ambiguity set. Assume that there are k nonoverlapping ambiguity sets for test points n_j, and F_i^j is the number of faults in ambiguity set S_i^j associated with test point n_j and integer code i. The probability of a fault being isolated from the ambiguity set S_i^j is approximated by F_i^j/f where f is the number of all potential faults listed in the dictionary. Suppose that the test points are independent with each other and the possibility for each fault happening is equal; therefore, the entropy-based measure, or information content $I(j)$ for the specific test point n_j is expressed by

$$\begin{aligned} I(j) &= -\left(\frac{F_{1j}}{f}\log\frac{F_{1j}}{f}+\frac{F_{2j}}{f}\log\frac{F_{2j}}{f}+\cdots+\frac{F_{kj}}{f}\log\frac{F_{kj}}{f}\right) \\ &= \frac{\log f}{f}\sum_{i=1}^{k}F_{ij}-\frac{1}{f}\sum_{i=1}^{k}F_{ij}\log F_{ij} \\ &= \log f-\frac{1}{f}\sum_{i=1}^{k}F_{ij}\log F_{ij} \qquad (1) \end{aligned}$$

where the entropy index $E(j)$ is defined as

$$E(j)=\sum_{i=1}^{f}F_{ij}\log F_{ij}. \qquad (2)$$

The physical explanation of $I(j)$ is the information used to be captured from imprecise knowledge that test point n_j contains.

Because the number of all faults in a given dictionary f is constant, the information content $I(j)$ for the specific test point n_j in (1) is maximized with the minimization of the entropy index $E(j)$. If a test point n_j with the minimum value of $E(j)$ is added to the desired test point set N_{opt} by inclusive approach, this will guarantee the maximal increase of information in N_{opt} by the maximal decrease of the entropy index. Consequently, this inclusive strategy guarantees that the maximal degree of fault diagnosis is achieved at each stage of test point inclusive selection.

The problem of searching for the appropriate test point for N_{opt} at any stage of the test points selection algorithm by inclusive approach is transformed to the problem of a linear search for a minimum value of $E(j)$, which can be easily and efficiently implemented.

A generalized algorithm for the proposed entropy-based method is given as follows.

Step 1) *Initialize the desired optimum set of test points* N_{opt} *as a null set.*

Step 2) *Calculate the number of faults in* F_i^j $(1 \le j \le t)$ *for the ambiguity set* S_i^j *and the test point* n_j.

Step 3) *Calculate the entropy index* $E(j)$ *by (2) for the test point* n_j *and search for the minimum value of* $E(j)$ *for each test point except for those already included in* N_{opt}.

Step 4) *Add test point* n_j *corresponding to the minimum value of* $E(j)$ *to the desired optimum set* N_{opt}. *The test point* n_j *will not be considered for future computation of entropy indices.*

Step 5) *If the minimum value of* $E(j)$ *is zero or if the new value of* $E(j)$ *is the same as the previous* $E(j)$ *for all test points, then stop.*

Step 6) *Partition the rows of the dictionary according to the ambiguity sets of* N_{opt} *and rearrange the dictionary by removing the rows whose size is unit in their partitions. Go to Step 2).*

Remark 1: In Step 6), if there are m ambiguity sets for the resulting optimum set N_{opt}, create m horizontal partitions of the dictionary.

Remark 2: If there is only one row in a partition in Step 6), the corresponding fault is concluded as uniquely isolated and should be removed from the dictionary. Thus, the size of ambiguity sets and the size of dictionary are gradually decreasing.

Remark 3: In Step 5), a maximum information increase to the N_{opt} is guaranteed each time by adding the test point n_j with minimum value of $E(j)$. Such an increase is a local maximal information increase at each stage of the algorithm. It is not indicated that the combination of all selected test points in N_{opt} will yield the maximum information. Global minimum set of test points can only be implemented by searching for the minimum value of the system entropy indices for exhaustive combinations of test points, which is expensive in computer resources and simulation time. The proposed entropy-based method is an appropriate candidate for an efficient selection technique of the optimum test points.

Most of methods for the optimum selection of test points reviewed in Section I [3], [6], [8]–[10] are not polynomial bounded. Although the method in [7] is polynomial bounded, it is not applicable for general cases. Among the polynomial-bounded algorithms in [13], the most efficient algorithm has the complexity of $O(fp\log f)$. The efficiency of the proposed entropy-based method is better than all of these methods because sorting is performed on ambiguity sets whose size is gradually reducing. Suppose that a dictionary has f rows and n columns. After the selection of one test point to N_{opt}, we obtain k smaller subdictionaries with $f_1, f_2, \ldots, f_k$ rows in each dictionary and $n-1$ columns where

$$\sum_{i=1}^{k} f_i \le f. \qquad (3)$$

Assuming that there are p test points totally examined during the selection process, the complexity of the sorting algorithm for the partition in Step 6) is

$$\begin{aligned} &O\left(p(f_1\log(f_1)+f_2\log(f_2)+\cdots+f_k\log(f_k))\right) \\ &< O\left(p(f_1\log f+f_2\log f+\cdots+f_k\log f)\right) \\ &< O\left(fp\log f\right). \qquad (4) \end{aligned}$$

Since the complexity of the proposed method is dominated by the complexity of the sorting algorithm, the overall complexity of the proposed method is less than the complexity of the best algorithm in [13] $O(fp\log f)$.

A similar entropy-based method has been presented in [19]. It exploits the concept of the information channel and minimization of the information deficit. The algorithm starts with the null set of test points. The channel inputs are circuit conditions $F=\{f_0, f_1, \ldots, f_f\}$. Ambiguity sets of the jth measurement $\mathrm{S}_j=\{\mathrm{S}_{j1}, \mathrm{S}_{j2}, \ldots\}$ are the outputs. In such a channel, misinformation is equal to zero, and, therefore, the obtained relative information is equal to the output entropy (information content), as given by (1). This concept leads to the identical entropy index $E(j)$ as given by (2), and the test point that gives the minimum index is selected at each consecutive step.

IV. EXPERIMENTS

The work on optimum selection of test points based on integer-coded dictionary by Prasad and Babu [13] is the latest comparison with the other work reported in literature. Therefore, three inclusive methods and one exclusive method pre-

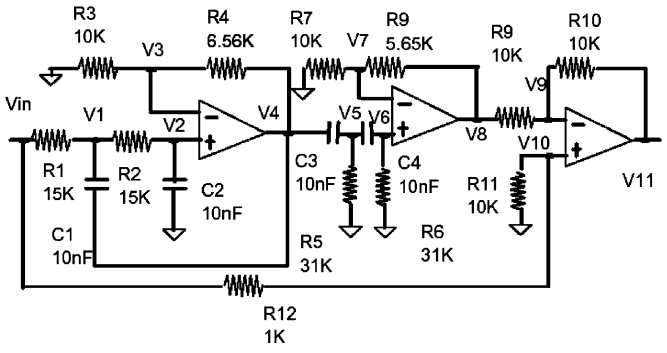

Fig. 1. Analog filter.

sented in [13] were selected in order to illustrate the computation efficiency and solution quality of the proposed entropy-based method.

A. Experiment on the Example of Analog Filter

The experiment is carried out on the active filter with the nominal parameter values as indicated in Fig. 1. This is the same example as in [13]. The excitation signal is a 1-kHz, 4-V sinusoidal wave. For simplification, 18 potential catastrophic faults are defined, and all 11 nodes are assumed to be accessible for the purpose of demonstration (this large number of test nodes seldom occurs in practice). The filter is then simulated by SABER. The integer-coded dictionary is constructed by procedures introduced in Section I based on simulated circuit responses, and is shown in Table II, which is also the same as Table I in [13]. f_0 is the nominal case while f_1-f_{18} are open or short faults. Test nodes n_1-n_{11} are the test points for selection. The experiment is programmed by MATLAB.

In Step 1) of the proposed entropy-based algorithm described in Section III, the desired optimum set of test points N_{opt} is initialized to be null. After calculating the size of each ambiguity set in Step 2), the entropy index $E(j)$ for each test node is calculated in Step 3) and illustrated in Table III. Node n_{11} has the minimum value of entropy index 6.6. Since the condition to stop in Step 5) is not satisfied, node n_{11} is selected and added to the desired optimum set and now $N_{\text{opt}} = \{n_{11}\}$. Node n_{11} will not be considered for the computation of entropy indices in the remaining iterations of the algorithm. For node n_{11}, nine ambiguity sets can be identified by Definition 2, as follows:

$$\begin{aligned}
S_0^{11} &= \{(f_2, f_{14})\,|a_{2,11} = a_{14,11} = 0\},\\
S_1^{11} &= \{(f_0, f_{18})\,|a_{0,11} = a_{18,11} = 1\},\\
S_2^{11} &= \{(f_4)\,|a_{4,11} = 2\},\ S_3^{11} = \{(f_{17})\,|a_{17,11} = 3\},\\
S_4^{11} &= \{(f_7, f_{11})\,|a_{7,11} = a_{11,11} = 4\},\ S_5^{11}=\{(f_5)\,|a_{5,11}=5\},\\
S_6^{11} &= \{(f_9)\,|a_{9,11} = 6\},\\
S_7^{11} &= \{(f_1, f_3, f_8, f_{10})\,|a_{1,11} = a_{3,11} = a_{8,11} = a_{10,11} = 7\},\\
S_8^{11} &= \{(f_6, f_{12}, f_{13}, f_{15}, f_{16})\,|a_{6,11} = a_{12,11} = a_{13,11}\\
&\quad = a_{15,11} = a_{16,11} = 8\}.
\end{aligned}$$

According to Step 6), dictionary in Table II is partitioned into nine subdictionaries determined by above nine ambiguity sets of N_{opt}. The result of partition is shown in Table IV.

TABLE II
INTEGER-CODED DICTIONARY FOR ANALOG FILTER

	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11
f0	3	2	2	3	3	3	4	4	1	1	1
f1	0	0	0	0	0	0	0	0	1	1	7
f2	3	2	2	3	4	3	4	4	1	1	0
f3	1	0	0	0	0	0	0	0	1	1	7
f4	2	3	3	4	6	5	6	6	1	1	2
f5	1	1	1	1	1	1	1	1	1	1	5
f6	0	0	0	2	3	2	3	3	1	1	8
f7	3	2	2	3	5	4	5	5	1	1	4
f8	3	2	2	3	0	0	0	0	1	1	7
f9	3	2	2	3	6	6	7	7	1	1	6
f10	3	2	2	3	2	0	0	0	1	1	7
f11	3	2	2	3	3	3	4	2	1	1	4
f12	3	2	2	3	3	2	0	8	5	1	8
f13	3	2	2	3	3	2	2	8	4	1	8
f14	3	2	2	3	3	3	4	4	1	1	0
f15	3	2	2	3	3	3	4	4	2	1	8
f16	3	2	2	3	3	3	4	4	3	1	8
f17	3	2	2	3	3	3	4	4	2	2	3
f18	3	2	2	3	3	3	4	4	0	0	1

TABLE III
RESULTS OF ENTROPY INDICES FOR THE FIRST ITERATION

Test nodes	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	**n_{11}**
Entropy Indexes E(j)	17.2	17.4	17.4	17.2	12.0	11.0	10.7	8.92	15.0	20.9	**6.6**

TABLE IV
PARTITIONED INTEGER-CODED DICTIONARY FOR THE FIRST ITERATION

	n_{11}	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}
f_2	0	3	2	2	3	4	3	4	4	1	1
f_{14}	0	3	2	2	3	3	3	4	4	1	1
f_0	1	3	2	2	3	3	3	4	4	1	1
f_{18}	1	3	2	2	3	3	3	4	4	0	0
f_4	2	2	3	3	4	6	5	6	6	1	1
f_{17}	3	3	2	2	3	3	3	4	4	2	2
f_7	4	3	2	2	3	5	4	5	5	1	1
f_{11}	4	3	2	2	3	3	3	4	2	1	1
f_5	5	1	1	1	1	1	1	1	1	1	1
f_9	6	3	2	2	3	6	6	7	7	1	1
f_1	7	0	0	0	0	0	0	0	0	1	1
f_3	7	1	0	0	0	0	0	0	0	1	1
f_8	7	3	2	2	3	0	0	0	0	1	1
f_{10}	7	3	2	2	3	2	0	0	0	1	1
f_6	8	0	0	0	2	3	2	3	3	1	1
f_{12}	8	3	2	2	3	3	2	0	8	5	1
f_{13}	8	3	2	2	3	3	2	2	8	4	1
f_{15}	8	3	2	2	3	3	3	4	4	2	1
f_{16}	8	3	2	2	3	3	3	4	4	3	1

By Remark 2, faults $\{f_4, f_{17}, f_5, f_9\}$ are concluded as uniquely isolated by measurements on node n_{11} and the corresponding rows, therefore, should be removed from Table IV.

Repeat computation of ambiguity set sizes and entropy indices on the updated dictionary. After partition and row removal, the resulting entropy indices are shown in Table V, in which node n_9 has the minimum value of $E(j)$, 3.61. Note that n_{11}

TABLE V
RESULTS OF ENTROPY INDICES FOR THE SECOND ITERATION

Test nodes	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	**n_9**	n_{10}
Entropy Indexes E(j)	4.81	5.42	5.42	5.42	5.53	5.65	4.21	4.82	**3.61**	7.71

TABLE VI
PARTITIONED INTEGER-CODED DICTIONARY AFTER THE SECOND ITERATION

	n11	n9	n1	n2	n3	n4	n5	n6	n7	n8	n10
f2	0	1	3	2	2	3	4	3	4	4	1
f14	0	1	3	2	2	3	3	3	4	4	1
f0	1	1	3	2	2	3	3	3	4	4	1
f18	1	0	3	2	2	3	3	3	4	4	0
f7	4	1	3	2	2	3	5	4	5	5	1
f11	4	1	3	2	2	3	3	3	4	2	1
f1	7	1	0	0	0	0	0	0	0	0	1
f3	7	1	1	0	0	0	0	0	0	0	1
f8	7	1	3	2	2	3	0	0	0	0	1
f10	7	1	3	2	2	3	2	0	0	0	1
f6	8	1	0	0	0	2	3	2	3	3	1
f12	8	5	3	2	2	3	3	2	0	8	1
f13	8	4	3	2	2	3	3	2	2	8	1
f15	8	2	3	2	2	3	3	3	4	4	1
f16	8	3	3	2	2	3	3	3	4	4	1

TABLE VII
RESULT OF ENTROPY INDICES FOR EACH ITERATION

Test nodes	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}
Entropy Indexes E(j)	17.3	17.5	17.5	17.3	12.0	11.1	10.7	8.93	15.1	20.9	**6.62**
	4.81	5.42	5.42	5.42	5.53	5.65	4.21	4.82	**3.61**	7.71	---
	1.80	2.40	2.40	2.40	**1.43**	3.01	3.01	3.01	---	3.61	---
	0.00	0.60	0.60	0.60	---	0.60	0.60	0.60	---	0.60	---

has already been excluded from the entropy index computation. Now, $N_{\text{opt}} = \{n_{11}, n_9\}$. There are ten ambiguity sets for test points n_{11} and n_9 together. The dictionary is partitioned according to these ten ambiguity sets as shown in Table VI. Faults $\{f_0, f_{18}, f_6, f_{12}, f_{13}, f_{15}, f_{16}\}$ can be uniquely isolated this time by Remark 2, and the corresponding rows should be removed from Table VI. The similar procedures continue as those outlined in the algorithm in Section III. The computed entropy indices for each iteration are shown in Table VII. The test points selected in each iteration are in bold. The algorithm stopped when the minimum value of $E(j)$ for n_1 is equal to zero which satisfies the condition for stop. The resulting optimum set of test nodes is $\{n_{11}, n_9, n_5, n_1\}$ which can fully isolate all the faults f_1-f_{18} in the given example.

As reported in [13], a set of nodes $\{n_{11}, n_8, n_7, n_5, n_9, n_1\}$ is found to be the final solution by using Algorithm 3 and Inclusive Strategy 2 in [13]. Obviously, the result obtained by inclusive method 2 in [13] is not optimum since nodes n_7 and n_8 are redundant. The computation time of the Algorithm 3 with Inclusive Strategy 2 in [13] is only slightly longer that the time required by our proposed algorithm. Using the Algorithm 4 and the exclusive Strategy 5 in [13], the final solution is found to consist of the set of nodes $\{n_1, n_5, n_9, n_{11}\}$ which is the same result as obtained by the proposed algorithm. However, the computation time of the exclusive algorithm in [13] is much longer. In addition, as it will be discussed in the following statistical experiments, the possibility of finding global minimum set of test points is much lower using the inclusive or exclusive algorithms in [13] than using the proposed entropy-based algorithm.

Except for computation efficiency and solution quality, another advantage of the proposed method is that it will not increase the ambiguities and complexity for the partially diagnosable system. Not all systems are fully diagnosable, especially for today's analog systems with the increasing die size, increasing integration complexity, and reduced accessibility. If the set of all the available test points, such as accessible test nodes, independent test frequencies, or valid sampling times, only describes a partially diagnosable system, the dictionary surely contains ambiguity sets whose elements cannot be uniquely distinguished. Under such conditions, the proposed method will reach an optimum subset of the set of all available test points without degrading the degree of system diagnosability. That is, the number of ambiguity sets and their complexity in the optimum set of selected test points stays the same as those in the set of all available test points.

B. Statistical Experiments

As discussed in Section III, the global minimum set of test points can only be guaranteed by an exhaustive search which is computationally expensive. Any efficient polynomial-bounded algorithm for test points selection only guarantees a local minimum solution. If no theoretical proof can be offered to demonstrate a specific nonexhaustive algorithm's optimality, such an algorithm must be tested statistically on large number of fault dictionaries in order to conclude its computation efficiency and qualities of the generated results.

Such statistical experiments were carried out on the randomly computer-generated integer-coded dictionaries by using the proposed entropy-based algorithm, three inclusive algorithms, and one exclusive algorithm in [13], respectively. Simultaneously, the exhaustive search algorithm based on rough set theory, as described in Section II, was also used here to provide a reference to the experimented algorithms. All the simulations are done by using MATLAB codes. Totally, there are 200 randomly computer-generated integer-coded dictionaries, and every dictionary includes 100 simulated faults, 30 test points, and five ambiguity sets per test point. These 200 dictionaries are first analyzed by the exhaustive search algorithm EXPANSION which generates all reducts of a given information system [16]. Minimum size reduct which corresponds to the global minimum set of test points is found by EXPANSION for each dictionary. The size of global minimum reduct is found to be 5 for all 200 cases. The same dictionaries are analyzed by using the proposed entropy-based method as well as three inclusive methods and one exclusive method in [13], respectively. The obtained statistical results concerning the solution accuracy are shown in Table VIII.

The conclusion from Table VIII is that the proposed method has significantly better quality in finding near-minimum solution. The global minimum sets of test points (size is 5) found by the proposed method were found in 35.5% of all the simulated cases, while the global minimum sets were found by the other algorithms in only between 0% and 1.5% of all simulated cases. Thus, the proposed entropy-based method has much higher possibility to find the global minimum set. Additionally, the pro-

TABLE VIII
STATISTICAL RESULTS OF THE SOLUTION ACCURACY

	Percentage of the optimum sets with a specific size found by a specific method					
Size of the optimum sets found	EXPANSION	Proposed Method	Exclusive Method	Inclusive Method 1	Inclusive Method 2	Inclusive Method 3
5	100	35.5	1.5	0	0.5	0.5
6	0	64.5	74.0	16.5	30.0	29.0
7	0	0	24.5	47.0	49.5	50.0
8	0	0	0	32.5	17.5	19.0
9	0	0	0	4.0	2.5	1.5

TABLE IX
COMPUTATION TIME PER DICTIONARY

	Proposed Method	Exclusive Method	Inclusive Method 1	Inclusive Method 2	Inclusive Method 3
Unit (seconds)	16.4	80.9	20.4	27.8	20.0

posed algorithm found 100% of the optimum sets of test points whose sizes are at most larger by 1 (size is 5 or 6) than the exact solution (size is 5). For the exclusive algorithm, 24.5% of the simulated cases are found to have the near-minimum sets whose sizes are larger by 2 (size is 7) than the exact solution. All the three inclusive algorithms are even worse, since about 70% or more of the simulated cases have the near-minimum sets of test points whose sizes are larger at least by 2 (size is 7, 8, or 9) than the exact solution.

Tested on a Pentium 586 PC with MATLAB 5.2, the average execution time per dictionary for the experimented algorithms is shown in Table IX. The conclusion is that the proposed algorithm is more computationally efficient since it took the shortest time (average 16.4 s) to finish one run on a single dictionary.

The scope and applicability of the proposed algorithm is also explored by comparing it with the exhaustive search algorithm. The relationship between the computation time and system complexity for the exhaustive search algorithm can be seen in [16, Fig. 1]. Computation time is proportional to the number of potential faults (number of signals in [16, Fig. 1]) and the number of available test points (number of attributes in [16, Fig. 1]). In addition, the larger the number of ambiguity sets per test point, the shorter the computation time of the exhaustive search algorithm. Therefore, the exhaustive search algorithm is limited to small or medium size analog systems. For large or medium systems, such as the dictionaries with more than 40 faults and more than 40 test points ([16, Fig. 1]), the exhaustive search is impractical. Therefore, the significance of the proposed entropy-based method is that it offers a test points selection method with high quality for large and medium size systems within a reasonable computational cost.

V. CONCLUSION

Optimum selection of test points is the most important stage in analog fault dictionary techniques. The global minimum solution is only guaranteed by exhaustive search which is NP-hard and, thus, is impractical for medium or large systems. The local minimum set is, therefore, the tradeoff to efficiently achieve the desired degree of fault diagnosis. Based on the integer-coded dictionary, an efficient method which finds a minimum test set is proposed in this paper. This method finds the minimum test set by using the entropy index of test points. Its complexity is proven to be less than $O(fp \log f)$. Carried out on the same trademark circuit, it shows better solution quality and higher computational efficiency than the other recently reported methods. Since no theoretical proof of quality can be given to the proposed method, statistical experiments are utilized for its evaluation. The simulated results demonstrate that the method is superior to other methods in its computational efficiency and quality of final solution; therefore, it is a good candidate for testing large scale systems.

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