Select(A, 1, n, i)
q = Partition(A, 1, n)

// q = n / 2

if (i = q)
    return A[i]
if (i < q)
    Select(A, 1, n / 2 - 1, i)
else
    Select(A, n/2 + 1, …)

Select: T(n) = T(n/2) + Theta(n) (“best” case) => T(n) = Theta(n)
Select: T(n) = T(n-1) + Theta(n) (worst case) => T(n) = Theta(n^2)

T(n) = a T(n/b) + f(n)

Master Theorem: n^(log_b (a)) vs. f(n)

n^log_b a = n^lg 2 = n^0 = 1 vs. Theta(n)

-------------------------------

QuickSort: T(n) = 2 T (n/2) + Theta(n) (“best” case)
QuickSort: T(n) = T(n-1) + Theta(n) (worst case) => Theta(n^2)

Imagine we have a way to find a pivot such that we recursively call Select on a fixed percentage of the input array. We “eliminate” a percentage $a$, we look only at $b = 1 – a$ percentage of elements in the input array.

Example, $b = 70\%$ of the elements in the recursive call.

T(n) = T(7/10 * n) + Theta(n) => by MT T(n) = Theta(n)

$n^0 = 1$ vs. $f(n) = \Theta(n)$ => MT says $T(n) = \text{linear}$.

______________________________

Matrix Multiplication:

A, B are $n \times n$ matrices of integers. Compute $C = A \cdot B$, what is $T(n)$ = ?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

$C[1, 1] = 1 \cdot 1 + 2 \cdot 1 = 3$
$C[1, 2] = 1 \cdot 1 + 2 \cdot 1 = 3$
Traditional algorithm:

for i=1 to n
    for j = 1 to n
        C[i,j] = 0
        for k=1 to n
            C[i,j] += A[i,k] * B[k,j]

\[ T(n) = n \cdot n \text{ (elements in C to compute) } \cdot \Theta(n) = \Theta(n^3) \]

Strassen showed how to do it in \( T(n^{2.8..}) = T(n^{\log_2 7}) \)

Can it ever be done in \( \Theta(n \log n) \)? No, it will be \( \Omega(n^3) \).

\[ T(n) = 8 \cdot T(n/2) + \Theta(n^2) \Rightarrow \text{MT Case 1} \quad T(n) = n^{\log_2 8} = n^3 \]

7 matrix multiplications \((n/2 \times n/2) \Rightarrow 7 \cdot T(n/2)\)

18 matrix additions \((n/2 \times n/2) \Rightarrow 18 \cdot n^2 / 4 = \Theta(n^3)\)

\[ T(n) = 7 \cdot T(n/2) + \Theta(n^2) \Rightarrow \text{by MT Case 1} \Rightarrow T(n) = \Theta(n^{\log_2 7})! \]