HW Assignment 3 (Due date: Feb 25, 9:00am)

1. [Divide and Conquer, 20 points] Provide a divide-and-conquer algorithm for determining the smallest and second smallest values in a given unordered set of numbers. Provide a recurrence equation expressing the time complexity of the algorithm, and derive its exact solution (i.e., not the asymptotic solution). For simplicity, you may assume the size of the problem to be an exact power of a natural number.

2. [Divide and Conquer, 20 points] Describe a divide-and-conquer algorithm for computing $a^n$ (where $n \in \mathbb{N}$) that runs in $\Theta(\log n)$ time. Justify your answer.

3. [Divide and Conquer, 20 points] You are given an array $A$ containing $n$ numbers. Design an $O(n \log n)$ Divide & Conquer algorithm for finding a pair of numbers $p$ and $q$ in $A$ such that $p$ appears before $q$ in $A$ and $q - p$ is maximum. Describe the algorithm and analyze its asymptotic time complexity.
   Example: if $A = 8 \ 1 \ 5 \ 0 \ 3 \ 9 \ 3 \ 7 \ 4$, the algorithm should return $p = 0$ and $q = 9$.

4. [Heap, 20 points] Exercise 6.4-4, page 160. For simplicity, you may assume the size of the problem to be an exact power of a natural number.

5. [Heap, 10 points] Give pseudocode for $\text{Heap-Decrease-Key}(A, i, key)$ that runs in $O(\log n)$ time for an $n$-element max-heap.

6. [QuickSort, 10 points] Exercise 7.2-1 page 178.

7. [QuickSort, 10 points] Write the pseudocode for the PARTITION algorithm studied in class.

8. (*) [Finding the missing integer, 20 points] An array $A[1..n]$ contains all the integers from 0 to $n$ except one. It would be easy to determine the missing integer in $O(n)$ time by using an auxiliary array $B[0..n]$ to record which numbers appear in $A$. In this problem, however, we cannot
access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is “fetch the jth bit of A[i]”, which takes constant time.

Show that if we use only this operation, we can still determine the missing integer in $O(n)$ time.