1. Proofs: Explain what is wrong with the following proof: We will prove by strong indiction that: \( \forall n \geq 0, a^n = 1 \), given that \( a > 0 \).
   Base case: \( a^0 = 1 \) which is true.
   Assume that \( a^n = 1 \) is true \( \forall n \mid 0 \leq n < m \)
   We must prove that \( a^m = 1 \)
   But we know: \( a^m = (a^{m-1})(a^{m-1})/(a^{m-2}) \).
   So by our inductive hypothesis, \( (a^{m-1}) = (a^{m-2}) = 1 \)
   Thus \( a^m = 1 * (1/1) = 1 \) Q.E.D.

2. Prove or disprove
   (a) \( f(n) \in O(f(n/4)) \)
   (b) \( n^2 \in o(n^3) \)
   (c) \( \sqrt{n} \in \Omega(n) \)
   (d) \( n^2 \in \omega(n) \)
   (e) \( 42n^2 + n \log n + 47 \in \Theta(n^2) \)

3. Provide a closed-form solution to the following:
   (a) \( \sum_{j=1}^{\infty} (\frac{1}{2})^j \)
   (b) \( \sum_{j=1}^{n} (2j + 2) \)

4. Solve the following recurrence relations
   (a) \( T(n) = T(2n/3) + 1 \)
   (b) \( T(n) = 2T(n/2) + n^2 \)
   (c) \( T(n) = 4T(n/2) + n \)
   (d) \( T(n) = T(n - 2) + n \)

5. T/F
   (a) \( \sqrt{n} \in \omega(\log n) \)
   (b) \( 2^n \in \Omega(2^{n+10}) \)
   (c) \( \sqrt{n} \in o(n) \)
   (d) \( n^{50} \in O(2^n) \)
6. Counting Sort

(a) Show the C matrix after the first and second steps of the algorithm with the following data: \(A = 2, 7, 4, 3, 5, 2, 1, 3, 1, 2, 4, 5, 7\)

(b) Show the contents of the B and C matrix after placing the first three values of A into B.

7. Selection: Given \(A = 15, 7, 9, 5, 12, 8, 7, 4, 11, 19, 13, 21, 6, 2, 18\)

(a) What are the medians found after the first pass?

(b) What is the median-of-medians that is finally found?